SC-RR-65-70

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SCALING DIMENSIONS OF CRATERS
PRODUCED BY BURIED EXPLOSIONS

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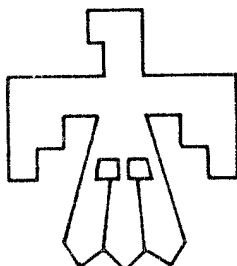
Sandia Corporation
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ALBUQUERQUE, NEW MEXICO
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SC-RR-65-70

SCALING DIMENSIONS OF CRATERS
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February 1965

ABSTRACT

The question of the proper scaling of crater dimensions resulting from buried explosions is investigated. Dimensional analyses are performed from which four different scaling rules are derived. Data are reviewed in an attempt to distinguish which scaling rules are fundamental to cratering. Inability to perform cratering experiments with similitude apparently is one reason for lack of an unambiguous answer to the scaling question. Influences of possible sources of similarity violation are qualitatively examined, and some experiments are suggested which may provide more direct information about the correct scaling of crater dimensions.

ACKNOWLEDGMENTS

Issued by Sandia Corporation,
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United States Atomic Energy Commission

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Chapter 1

INTRODUCTION

One of the conspicuous peaceful applications of nuclear explosives is that of earth excavation, as might be considered for the construction of harbors, dams, or canals. Since the energy release of nuclear explosives can be fantastically great, the amount of excavation or size of a crater resulting from nuclear explosions, is for many engineering applications, practically unlimited.

For single spherical explosions, the size of the crater formed depends principally on the energy release and depth of burial of the explosive, and to a lesser but important extent on the properties of the earth medium in which the explosion occurs.

Our ability to accurately predict crater sizes from large nuclear detonations is at present uncertain because of lack of adequate theories and particularly because of lack of experimental data from large explosions.

To estimate crater dimensions resulting from large nuclear bursts, a common procedure is to use available data from explosions in the medium of interest and to scale these crater dimensions to the much larger energies of nuclear explosions. Such a procedure may involve a considerable extrapolation of the particular scaling laws used, and may, as in any uncertain extrapolation, give rise to large errors. The procedure is also often complicated by insertion of largely unknown and possibly meaningless "efficiencies" relating nuclear to chemical explosives.

The most recent nuclear cratering experiment, Sedan (100 kilotons), has provided valuable data which alleviate to some extent the difficulty of extrapolating to megaton-size explosions, particularly in soil media. Data from Sedan, however, are not sufficiently definitive to answer unambiguously the basic and related questions of scaling and relative cratering effectiveness of nuclear, as compared to chemical, explosions.

For a period of years following Lampson's work[1] and the work of Morrey, et al.[2], it was generally accepted that crater dimensions resulting from buried spherical explosions are proportional to, or are scaled by, the cube root of weight of explosive. This conclusion is based on the argument that the characteristic length in

experiments is the radius of the (spherical) explosive charge, which length is proportional to the cube root of the charge weight. Thus, explosions of various energies buried at the same scaled burst depth (depth divided by the cube root of weight of explosive) would produce the same scaled crater dimensions (linear dimension divided by the cube root of weight of explosive). Over the small range of explosion energies investigated, cube-root scaling was confirmed by experimental data.[3]

As more crater data became available from explosions of still greater energy release, some hints[4,5,6] were presented that possibly cube-root scaling was not strictly valid over a larger range of explosion energies. Largely because insufficient data were obtained, because scatter in data was usually great, because data obtained were not from controlled experiments designed to examine cratering, or for a number of other reasons, these first indications of the limited applicability of cube-root scaling were, understandably, largely ignored, and more or less explicit faith in cube-root rules was maintained.

A regression analysis[7] of extensive crater data from explosions in Nevada desert alluvium provided the result that these data were not properly scaled by cube-root rules. Instead of being proportional to the cube root of explosion energy, linear crater dimensions were found to be more nearly proportional to the 3/10 power of energy. The average standard deviation associated with the empirically derived 3/10 power was 0.02 for linear crater dimensions, i.e., 0.296 ± 0.024 (0.291 ± 0.016 for radius, 0.298 ± 0.036 for depth, and 0.299 ± 0.021 for $volume^{1/3}$). More recent data[8,9] from explosions of 20 tons and 500 tons of TNT in desert alluvium have reduced this average standard deviation to 0.019, i.e., 0.296 ± 0.019, which result is considered a confirmation that cube-root scaling is not correct over the range of explosive weights from 10^2 to 10^6 pounds of TNT. Independent examinations of desert alluvium data by Vaile,[10] Violet,[11] and Nordyke[12] have also resulted in the conclusion that 3/10 rules scale desert alluvium data better than cube-root rules.

This apparently small difference, 1/3 versus 3/10, in the power of explosion energy by which dimensions scale is negligible for experiments with small explosives (less than 10 pounds TNT) whose energy ratios are 10 or less, and is in accord with the earlier observations of Lampson and others from model experiments that cube-root rules are adequate for scaling over this energy range. In experiments with energies whose ratios lie between 10 and 100 (about 10 to 10^3 pounds TNT), inherent scatter in the data is generally sufficient to obscure the deviation from cube-root rules. For explosion energies with ratios greater than 100, deviations from cube-root rules exceed inherent scatter and become observable. When energy ratios are larger than 10^3, the difference in crater dimensions obtained from 3/10 and 1/3 scaling rules becomes appreciable, and points out one significant difficulty in attempting to scale chemical-explosive results to energies of large nuclear explosives.

Experimental crater data from explosions at the Nevada Test Site in desert alluvium have cast doubt on our ability to scale accurately crater dimensions in any geologic medium. Since cube-root rules are incorrect for craters in desert alluvium, they may well be inadequate for craters in any other medium over an arbitrary range of explosion-energy values. The empirical 3/10 scaling rule appears adequate for chemical explosions in desert alluvium over the range of energy release from 10^2 to 10^6 pounds of TNT, but there is at present no reason for having confidence that this empirical rule will suffice for scaling to the energies of much larger nuclear explosions. An additional complication in attempting to scale crater dimensions to energies of nuclear explosions in media other than desert alluvium rests in the fact that the 3/10 scaling rule has not been definitely established for any medium other than desert alluvium. However, in view of the inadequacy of cube-root rules, a tendency exists to assume the validity of 3/10 rules for any medium. Acceptance of this assumption, coupled with the requirement of extrapolation to large nuclear explosion energies, may well lead to estimates of nuclear-explosion crater dimensions which are grossly in error.

Explanations advanced to account for the inadequacy of cube-root scaling are generally qualitative and involve arguments concerning lack of homogeneity or anisotropy of the test medium, or the influence of gravitational acceleration.[13] Few attempts[5,8,14] have been made to actually scale data, taking any of these possible causes into account.

Since there are at present no complete theories of cratering from buried explosions, we shall inquire into the phenomena of cratering by means of dimensional analysis with the aim of finding some reasonable explanation for the lack of validity of cube-root rules observed in desert alluvium, of finding at least a qualitative explanation for the empirical 3/10 scaling rule, and with the objective of learning under what conditions crater dimensions can be accurately scaled in any medium, particularly from model studies, to the large energies of nuclear explosions.

In the next chapter, dimensional analyses are performed in which the acceleration of gravity is both included and excluded, and in which the explosive source is described by a mass and by an energy dimension. Scaling results thus derived are compared with experimental data in Chapter 4 in an effort to determine which, if any, of the possible scaling rules best describe the data.

Only crater data from experiments with buried explosions in an alluvial soil at the Nevada Test Site have been considered, since they represent the most extensive and reliable set of data for any medium, including results from both nuclear and chemical explosions. Cratering from surface or above-ground-level bursts is not considered because of the paucity of data. Likewise, cratering from explosions in the vicinity of containment burial depths is not considered.

Chapter 2

DIMENSIONAL ANALYSIS

In the following analysis, assumptions are made that the medium in which a crater is formed by a buried explosion is homogeneous and isotropic.

Consider the following physical quantities (dimensions in parentheses) as being sufficient to describe the phenomena of cratering:

Medium Properties

ρ, (ML^{-3}), density of undisturbed medium.
Y, $(ML^{-1}T^{-2})$, a yield strength of the medium.
ν, $(ML^{-1}T^{-1})$, a viscosity or dissipation variable of medium.
c, (LT^{-1}), sonic velocity in the medium.

Independent Variables

a, (L), radius of spherical explosive charge.
d, (L), depth of burial of explosive charge.
p, $(ML^{-1}T^{-2})$, a hydrostatic pressure.
W, (M) or E, $(ML^{2}T^{-2})$, mass or energy of explosive charge.
R, (L), distance from explosion center.
t, (T), time after explosion.

Dependent Variables

r, (L), crater radius.
h, (L), crater depth.
V, (L^{3}), crater volume.
u, (LT^{-1}), velocity of medium particle.
α, (LT^{-2}), acceleration of medium particle.
σ, $(ML^{-1}T^{-2})$, stress on medium particle.
τ, (T), characteristic time, wave period.

Constants

g, (LT^{-2}), acceleration of gravity.

Other variables could also be included. Dimensionless quantities such as strain, void ratio, or moisture content of the medium can simply be inserted into the final result (see Equations 1 through 4) and do not affect the dimensional

analysis on the listed variables. Quantities such as a velocity, v, which have dimensions identical to a listed variable, appear in the final result as dimensionless ratios, such as v/c.

Any other variables which are considered significant to cratering and whose dimensions involve mass, length, and time only, may be added to the list and their insertion made in the final result without difficulty.

Although it is not definitely known whether the yield strength (compressive or tensile), Y, is significant to cratering phenomena in soils, it has been included for generality. For <u>continuous</u> rock media, one expects this variable to be important, since it has been shown[15] that the mechanism of crater formation in rock involves a spallation process into which tensile yield strength enters.

Similarly, for the sake of generality, a dissipation variable, v, in the form of a viscosity has been assumed important in the process of crater formation. If these variables are not significant to cratering, they can be discarded from consideration after completion of the dimensional analysis. Omission of variables, should they be significant, leads to erroneous conclusions. The relevance of yield strength and viscosity must ultimately be determined by experiment. In any event, results of a dimensional analysis will allow their influence in crater scaling to be examined qualitatively.

While the meaning of those quantities which we have assigned to describe medium properties, mass or energy of the explosive, and gravitational acceleration is precise, some ambiguity exists in the interpretation of those dependent variables, u, α, and τ, which are intended to describe crater dynamics. For example, it is not apparent initially whether u and α are the velocity and acceleration of medium particles being ejected from the region which becomes the crater void or whether they are particle motions resulting from the stress wave transmitted to the medium by the explosion. (There is ample evidence[16,17] to show that these two types of motion are different and quite distinct.) Also the variable, p, is subject to several interpretations such as atmospheric pressure, lithostatic pressure, or their sum. These ambiguities are characteristic in a dimensional analysis of complex phenomena such as cratering, where the complete differential equations and the medium constitutive relations which describe the phenomena are not known. The validity of interpretation assigned to variables entering a dimensional analysis can only be justified by the reasonableness of conclusions drawn from the analysis and by experimental verification.

Since we are interested in the mechanisms of crater formation, it is desirable that the variables, u and α, refer to velocity and acceleration experienced by a soil particle which is ejected or moved far from its original position by an explosion, and not to velocity and acceleration imparted to a soil particle by shock or seismic waves. The variables are to be correlated with motion of upheaved earth, such as that considered by Knox,[18] at times long compared to shock or elastic-wave

arrival time at the ground surface and to the motion of missiles that are discharged from the region of earth which eventually becomes the crater void. The variable, t, describes the time of an event in crater formation, and τ should be representative of a characteristic time or period related to crater formation and not to the period of seismic waves, the shock-wave decay time, or the arrival time of shock or seismic waves at a given position.

By means of a dimensional analysis[19] performed on the variables listed, the following general relationships are obtained when W rather than E is used to describe the explosive, and gravity, g, is not included in the analysis.

$$r\left(\frac{\rho}{W}\right)^{1/3} = F_1\left\{d\left(\frac{\rho}{W}\right)^{1/3},\ a\left(\frac{\rho}{W}\right)^{1/3},\ \frac{Y}{\rho c^2},\ \frac{\nu}{\left(W\rho^2 c^3\right)^{1/3}},\ \frac{p}{\rho c^2},\right.$$

$$\left. R\left(\frac{\rho}{W}\right)^{1/3},\ ct\left(\frac{\rho}{W}\right)^{1/3},\ \cdots\right\} \tag{1a}$$

$$h\left(\frac{\rho}{W}\right)^{1/3} = F_2 \qquad\qquad \frac{\alpha}{c^2}\left(\frac{W}{\rho}\right)^{1/3} = F_5$$

$$v\left(\frac{\rho}{W}\right) = F_3 \qquad\qquad \frac{u}{c} = F_6 \tag{1b}$$

$$\frac{\sigma}{\rho c^2} = F_4 \qquad\qquad c\tau\left(\frac{\rho}{W}\right)^{1/3} = F_7$$

The arguments of the function F_1 are dimensionless quantities, and the dots indicate where other variables in appropriate dimensionless form may be included, e.g., strain; moisture content; velocities, v, in the vorm v/c; pressures, P, as P/p; etc. Arguments of functions F_2 through F_7 are identical to those of F_1.

To the list of quantities used to obtain Equation 1, let us add the acceleration of gravity, g, and again perform a dimensional analysis.

The results with g included, and again using W to describe the explosive, are:

$$r\left(\frac{\rho}{W}\right)^{1/3} = G_1\left\{d\left(\frac{\rho}{W}\right)^{1/3},\ a\left(\frac{\rho}{W}\right)^{1/3},\ \frac{Y}{\rho c^2},\ \frac{\nu}{\left(W\rho^2 c^3\right)^{1/3}},\ \frac{p}{\rho c^2},\right.$$

$$\left. R\left(\frac{\rho}{W}\right)^{1/3},\ ct\left(\frac{\rho}{W}\right)^{1/3},\ \frac{c^2}{gd},\ \cdots\right\} \tag{2a}$$

$$h\left(\frac{\rho}{W}\right)^{1/3} = G_2 \qquad\qquad \frac{\alpha}{c^2}\left(\frac{W}{\rho}\right)^{1/3} = G_5$$

$$V\left(\frac{\rho}{W}\right) = G_3 \qquad\qquad \frac{u}{c} = G_6 \qquad\qquad (2b)$$

$$\frac{\sigma}{\rho c^2} = G_4 \qquad\qquad c\tau\left(\frac{\rho}{W}\right)^{1/3} = G_7$$

If a dimensional analysis is now performed on the quantities listed with g excluded and with the mass of explosive, W, (M), replaced by the explosion energy, E, (ML^2T^{-2}), one obtains:

$$r\left(\frac{\rho c^2}{E}\right)^{1/3} = H_1\left\{ d\left(\frac{\rho c^2}{E}\right)^{1/3}, \; a\left(\frac{\rho c^2}{E}\right)^{1/3}, \; \frac{Y}{\rho c^2}, \; \frac{\nu}{\left(E\rho^2 c\right)^{1/3}}, \right.$$

$$\left. \frac{p}{\rho c^2}, \; R\left(\frac{\rho c^2}{E}\right)^{1/3}, \; t\left(\frac{\rho c^5}{E}\right)^{1/3}, \; \ldots \right\} \qquad (3a)$$

$$h\left(\frac{\rho c^2}{E}\right)^{1/3} = H_2 \qquad\qquad \alpha\left(\frac{E}{\rho c^8}\right)^{1/3} = H_5$$

$$V\left(\frac{\rho c^2}{E}\right) = H_3 \qquad\qquad \frac{u}{c} = H_6 \qquad\qquad (3b)$$

$$-\frac{\sigma}{\rho c^2} = H_4 \qquad\qquad \tau\left(\frac{\rho c^5}{E}\right)^{1/3} = H_7$$

Including g, and again using E in place of W, yields:

$$r\left(\frac{\rho c^2}{E}\right)^{1/3} = I_1\left\{ d\left(\frac{\rho c^2}{E}\right)^{1/3}, \; a\left(\frac{\rho c^2}{E}\right)^{1/3}, \; \frac{Y}{\rho c^2}, \; \frac{\nu}{\left(E\rho^2 c\right)^{1/3}}, \right.$$

$$\left. \frac{p}{\rho c^2}, \; R\left(\frac{\rho c^2}{E}\right)^{1/3}, \; t\left(\frac{\rho c^5}{E}\right)^{1/3}, \; \frac{c^2}{gd}, \; \ldots \right\} \qquad (4a)$$

$$h\left(\frac{\rho c^2}{E}\right)^{1/3} = I_2 \qquad \alpha\left(\frac{E}{\rho c^8}\right)^{1/3} = I_5$$

$$V\left(\frac{\rho c^2}{E}\right) = I_3 \qquad \frac{u}{c} = I_6 \qquad\qquad (4b)$$

$$\frac{\sigma}{\rho c^2} = I_4 \qquad \tau\left(\frac{\rho c^5}{E}\right)^{1/3} = I_7$$

At first sight it may appear superfluous to perform a dimensional analysis with the explosive source described by a mass dimension, W, and also with an energy dimension, E, since the energy release of any given chemical explosive is directly proportional to its mass. Nevertheless, it is not clear a priori that dimensional analyses with W and with E should lead to the same scaling results. Indeed, since we are considering a number of physical quantities in the analysis, some or all of the dimensionless variables resulting from the analysis might be expected to have different forms and hence different scaling rules, depending on whether W or E is considered, simply because the dimensions of W and E are different. From Equations 1 through 4, this is seen to be the case.

Equations 1 through 4 reveal how the physical quantities involved must be altered in order to perform model or small-scale experiments. The model experiment is said to be similar to the prototype only if all dimensionless quantities in each equation have the same numerical value for both experiments. If one or more dimensionless quantities are not numerically equal for both experiments, then similarity does not exist and the model experiment will not exactly duplicate phenomena of the prototype experiment. Dimensional analysis does not provide the analytical form of the functions F, G, H, and I in Equations 1 through 4. Without a theory, the function must be determined by experiment.

Consider Equation 1a and examine the manner in which a crater radius, r, is scaled by this equation.

If two cratering experiments with explosives of mass W_1 and W_2 are performed, and if similarity exists, the ratio of resulting crater dimensions, r_1 and r_2, will be:

$$\frac{r_1}{r_2} = \left(\frac{\rho_2}{\rho_1}\right)^{1/3} \cdot \left(\frac{W_1}{W_2}\right)^{1/3} \qquad\qquad (5)$$

Because the two experiments are similar, the arguments of F_1 in Equation 1a are numerically equal for both experiments; i.e.,

$$d_1/d_2 = R_1/R_2 = a_1/a_2 = (\rho_2/\rho_1)^{1/3}(W_1/W_2)^{1/3},$$

$$Y_1/Y_2 = p_1/p_2 = \rho_1 c_1^2 \Big/ \rho_2 c_2^2, \quad \nu_1/\nu_2 = \left(\rho_1^2 c_1^3 W_1 \Big/ \rho_2^2 c_2^3 W_2\right)^{1/3}, \text{ and}$$

$$t_1/t_2 = \left(\rho_2 c_2^3 W_1 \Big/ \rho_1 c_1^3 W_2\right)^{1/3}.$$

Consequently, the function F_1 is identical for both experiments and Equation 5 follows. Equation 5 is the well-known cube-root "mass" scaling rule of Lampson.[1] Note that the medium properties, ρ, c, and Y, need not be scaled, but may remain constant in two experiments without violating the similarity requirement. Also the hydrostatic pressure, e.g., atmospheric pressure acting on the ground surface above the cratering explosion, need not be scaled. The medium viscosity, ν, however, must be scaled in order to achieve similitude for the two experiments if ρ and c are kept constant. Otherwise, the scaling rule of Equation 5 is no longer strictly valid.

Similarly, from Equation 3a, two similar explosions of energies E_1 and E_2 will give crater radii whose ratio is

$$\frac{r_1}{r_2} = \left(\frac{\rho_2}{\rho_1}\right)^{1/3}\left(\frac{c_2}{c_1}\right)^{2/3}\left(\frac{E_1}{E_2}\right)^{1/3} \tag{6}$$

provided

$$d_1/d_2 = R_1/R_2 = a_1/a_2 = (\rho_2/\rho_1)^{1/3}(c_2/c_1)^{2/3}(E_1/E_2)^{1/3},$$

$$Y_1/Y_2 = p_1/p_2 = \rho_1 c_1^2 \Big/ \rho_2 c_2^2, \quad \nu_1/\nu_2 = \left(\rho_1^2 c_1 E_1 \Big/ \rho_2^2 c_2 E_2\right)^{1/3}, \text{ and}$$

$$t_1/t_2 = \left(\rho_2 c_2^5 E_1 \Big/ \rho_1 c_1^5 E_2\right)^{1/3}.$$

As for Equations 1a and 1b, we see that ν must be scaled to insure similarity among experiments and that ρ, c, Y, and p may remain constant. The scaling rule of Equation 6 was first obtained by Sachs[20] from consideration of blast waves in air.

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If, as is usually the case, cratering explosions occur in a medium where ρ and c are constant, Equations 5 and 6 are identical and scaling of crater dimensions by the mass of explosive charge, W, is equivalent to scaling by the energy, E. However, if experiments were performed in two media where only sonic velocities, c_1 and c_2, were not equal, then Equations 5 and 6 would lead to different results, the ratio of crater radii being different by a factor of $(c_2/c_1)^{2/3}$. Only if experiments could be performed in two media in which sonic velocities are not identical, but which are otherwise identical, could a distinction be made between the "mass" scaling rule of Equation 5 and the "energy" scaling rule of Equation 6. We see that the substitution $W = E/c^2$ makes Equation 1a identical to Equation 3a. So for similar experiments in a medium where c is constant and need not be scaled, Equations 1 and 3 yield the same scaling rule. Thus, <u>when the acceleration of gravity, g, is not considered significant, cube-root rules result for both "mass" and "energy" scaling</u>.

Experiments by Ericsson and Edin[21] were conducted in an effort to make a distinction between scaling rules of Equations 1 and 3 for blast waves in air. Explosions were detonated in air at different temperatures so that c was not constant for all experiments. Their results show that the "mass" scaling rule (Equation 1) is not correct, but that the "energy" scaling rule (Equation 3) is correct within the limits of experimental error. As a result of Ericsson and Edin's work, we might well conclude that, in a dimensional analysis which attempts to establish scaling rules for buried explosions, the explosive source should be described by an energy dimension rather than a mass dimension. However, in our next consideration of scaling rules which result when acceleration of gravity is included in the dimensional analysis, we examine results for both a mass and energy dimension describing the explosion source. Even though Ericsson and Edin's verification of "energy" scaling appears to be definitive and can probably be extended to buried explosions, the "mass" scaling rules, when g is included, are examined for heuristic purposes.

Considering next the scaling rules resulting from dimensional analysis with gravity included, we obtain from Equation 2a

$$\frac{r_1}{r_2} = \left(\frac{\rho_2}{\rho_1}\right)^{1/3}\left(\frac{W_1}{W_2}\right)^{1/3} \tag{7}$$

which is identical to the expression of Equation 5. For similitude between the two explosions, we require that

$$d_1/d_2 = R_1/R_2 = a_1/a_2 = (\rho_2/\rho_1)^{1/3}(W_1/W_2)^{1/3}, \quad Y_1/Y_2 = P_1/P_2 =$$

$$\rho_1 c_1^2 / \rho_2 c_2^2 = \rho_1 g_1 d_1 / \rho_2 g_2 d_2 = (g_1/g_2)(\rho_1/\rho_2)^{2/3}(W_1/W_2)^{1/3},$$

$$\nu_1/\nu_2 = \left(\rho_1^2 c_1^3 W_1 / \rho_2^2 c_2^3 W_2\right)^{1/3} = (g_1/g_2)^{1/2}(\rho_1/\rho_2)^{1/2}(W_1/W_2)^{1/2},$$

$$t_1/t_2 = \left(\rho_2 c_2^3 W_1 / \rho_1 c_1^3 W_2\right)^{1/3} = (g_2/g_1)^{1/2}(\rho_2/\rho_1)^{1/6}(W_1/W_2)^{1/6}, \text{ and}$$

$$c_1/c_2 = (g_1/g_2)^{1/2}(\rho_2/\rho_1)^{1/6}(W_1/W_2)^{1/6}.$$

To insure the validity of Equation 7, medium properties, ν, Y, and c, must be scaled as indicated and also hydrostatic pressure, p, must be scaled when ρ and g remain constant in experiments.

From Equation 4a we obtain the well-known fourth-root scaling rule[5,13]

$$\frac{r_1}{r_2} = \left(\frac{\rho_2}{\rho_1}\right)^{1/3}\left(\frac{c_2}{c_1}\right)^{2/3}\left(\frac{E_1}{E_2}\right)^{1/3} = \left(\frac{\rho_2 g_2}{\rho_1 g_1}\right)^{1/4}\left(\frac{E_1}{E_2}\right)^{1/4} \tag{8}$$

and

$$d_1/d_2 = R_1/R_2 = a_1/a_2 = (\rho_2 g_2/\rho_1 g_1)^{1/4}(E_1/E_2)^{1/4}, \quad Y_1/Y_2 =$$

$$P_1/P_2 = \rho_1 c_1^2 / \rho_2 c_2^2 = \rho_1 g_1 d_1 / \rho_2 g_2 d_2 = (\rho_1 g_1/\rho_2 g_2)^{3/4}(E_1/E_2)^{1/4},$$

$$\nu_1/\nu_2 = \left(\rho_1^5 g_1 / \rho_2^5 g_2\right)^{1/8}(E_1/E_2)^{3/8}, \quad t_1/t_2 = \left(\rho_2 g_2^5 / \rho_1 g_1^5\right)^{1/8}(E_1/E_2)^{1/8} \text{ and}$$

$$c_1/c_2 = \left(\rho_2 g_1^3 / \rho_1 g_2^3\right)^{1/8}(E_1/E_2)^{1/8},$$

from which it is seen that ν, Y, c, and p must be scaled when ρ and g are constant.

Note also that the charge radius, a, must be scaled by the fourth root of charge energy. Since $E = qW = (4/3)\pi\rho_x qa^3$ (q = energy release per unit mass of explosive, ρ_x = density of explosive), $a \propto E^{1/3}$ in experiments with a given explosive which is a violation of the similarity requirement in fourth-root scaling.

Equations 2a and 4a may be written in a more informative way as

$$r\left(\frac{\rho}{W}\right)^{1/3} = G_1 \left\{ d\left(\frac{\rho}{W}\right)^{1/3}, \ a\left(\frac{\rho}{W}\right)^{1/3}, \ \frac{Y}{(\rho^2 g^3 W)^{1/3}}, \ \frac{\nu}{(\rho g W)^{1/2}}, \right.$$

(2a')

$$\left. \frac{p}{(\rho^2 g^3 W)^{1/3}}, \ R\left(\frac{\rho}{W}\right)^{1/3}, \ t\left(\frac{\rho g^3}{W}\right)^{1/6}, \ c\left(\frac{\rho}{g^3 W}\right)^{1/6}, \ \ldots \right\}$$

$$r\left(\frac{\rho g}{E}\right)^{1/4} = I_1 \left\{ d\left(\frac{\rho g}{E}\right)^{1/4}, \ a\left(\frac{\rho g}{E}\right)^{1/4}, \ \frac{Y}{(\rho^3 g^3 E)^{1/4}}, \ \frac{\nu}{(\rho^5 g E^3)^{1/8}}, \right.$$

(4a')

$$\left. \frac{p}{(\rho^3 g^3 E)^{1/4}}, \ R\left(\frac{\rho g}{E}\right)^{1/4}, \ t\left(\frac{\rho g^5}{E}\right)^{1/8}, \ c\left(\frac{\rho}{g^3 E}\right)^{1/8}, \ \ldots \right\}$$

The substitution $c^2 W = E$ does not render Equations 2 and 4 identical as it did for Equations 1 and 3. The reason for this is that when gravity is included in the dimensional analysis, the medium acoustic velocity must be scaled so that W is not directly proportional to E as it is when gravity is omitted from the analysis.

To distinguish between the "mass" and "energy" scaling rules obtained from Equations 1 and 2 and those obtained from Equations 2 and 4, we shall refer to rules from Equation 2 as "mass-gravity" and to those of Equation 4 as "energy-gravity" scaling rules.

In Equations 1 and 3, the hydrostatic pressure, p, may be interpreted as atmospheric pressure. In Equations 2 and 4, p may be interpreted as lithostatic pressure, $\rho g d$, atmospheric pressure, or the sum of lithostatic and atmospheric pressures. The term p in Equations 1 and 3 cannot be considered lithostatic pressure, for then it would be recognized that gravity, g, was omitted from the dimensional analysis.

If $p = \rho g(d + k)$, where $k = p_{atm}/\rho g$ is that depth of earth equivalent to atmospheric pressure, then (with ρ and g constant) $p_1/p_2 = (d_1 + k_1)/(d_2 + k_2) = k_1/k_2 = d_1/d_2$ must be in the ratio $(W_1/W_2)^{1/3}$ or $(E_1/E_2)^{1/4}$, depending on whether cube-root scaling of Equation 2a' or fourth-root scaling of Equation 4a' is accepted. Thus, to insure similarity, atmospheric pressure must also be scaled if it is considered significant.

From Equations 1 through 4, scale factors for all the physical variables considered may be obtained as has been done for the variable r. The scale factors are expressed in terms of ratios of explosion masses or energies. They indicate the ratios that must be maintained among the independent variables for the realization of similarity and also the ratios that will result among the dependent variables for any two similar experiments with different amounts of explosives. These scale factors are listed in Table 1 which is compiled with the assumption that ρ and g are always constant. The ratio of charge masses is $S = W_1/W_2$ and the ratio of energies is $\Sigma = E_1/E_2$. Note that the relationship, $\Sigma = E_1/E_2$, may be expressed as $\Sigma = W_1/W_2$ only now that the dimensional analysis is complete.

In most cratering experiments it is not practical or possible to scale medium properties such as Y and v to insure similarity. Since these medium properties are not scaled but remain constant in experiments, they violate the requirement of similarity and hence make scaling rules invalid or only approximately correct. Parentheses have been placed around scale factors in Table 1 to indicate those quantities which usually contribute to violation of similarity in experiments.

From Table 1, the following observations may be made:

1. When acceleration of gravity is not considered significant, crater dimensions scale by the cube-root rule.

2. When acceleration of gravity is considered significant and is included in the dimensional analysis, crater dimensions are scaled by cube-root rules with "mass-gravity" scaling, and by fourth-root rules with "energy-gravity" scaling.

3. While dimensional analysis, excluding gravity, yields the result that stress and velocity fields are invariant in similar experiments at the same scaled distances and times, when gravity is included, the analysis reveals that only the acceleration field is invariant at identical scaled times and distances in similar experiments.

4. For mass or energy scaling without gravity, velocities are invariant and times are scaled by the cube root of explosive mass or energy. For "mass-gravity" scaling, velocities and times are scaled by the sixth root of the explosive mass, whereas for "energy-gravity" scaling, velocities and times are scaled by the eighth root of explosion energy.

5. When acceleration of gravity is not considered significant, the only medium property in mass or energy scaling which contributes to violate similarity in usual experiments is viscosity, v, which must be scaled by $W^{1/3}$ or $E^{1/3}$ if similarity is to be realized.

TABLE 1

Factors of Different Scaling Rules for Scaling
Variables in Similar Cratering Experiments
When ρ and g Are Constant

Ratio	Without g		With g	
	Mass scaling (Lampson)	Energy scaling (Sachs)	Mass-gravity scaling	Energy-gravity scaling (Haskell)
Dependent variables				
r_1/r_2	$s^{1/3}$	$\Sigma^{1/3}$	$s^{1/3}$	$\Sigma^{1/4}$
h_1/h_2	$s^{1/3}$	$\Sigma^{1/3}$	$s^{1/3}$	$\Sigma^{1/4}$
V_1/V_2	s	Σ	s	$\Sigma^{3/4}$
u_1/u_2	1	1	$s^{1/6}$	$\Sigma^{1/8}$
a_1/a_2	$s^{-1/3}$	$\Sigma^{-1/3}$	1	1
σ_1/σ_2	1	1	$s^{1/3}$	$\Sigma^{1/4}$
τ_1/τ_2	$s^{1/3}$	$\Sigma^{1/3}$	$s^{1/6}$	$\Sigma^{1/8}$
Variables of medium and explosive				
a_1/a_2	$s^{1/3}$	$\Sigma^{1/3}$	$s^{1/3}$	$(\Sigma^{1/4})$
Y_1/Y_2	1	1	$(s^{1/3})$	$(\Sigma^{1/4})$
ν_1/ν_2	$(s^{1/3})$	$(\Sigma^{1/3})$	$(s^{1/2})$	$(\Sigma^{3/8})$
c_1/c_2	1	1	$(s^{1/6})$	$(\Sigma^{1/8})$
Strain	1	1	1	1
Void ratio	1	1	1	1
Moisture content	1	1	1	1
Independent variables				
d_1/d_2	$s^{1/3}$	$\Sigma^{1/3}$	$s^{1/3}$	$\Sigma^{1/4}$
R_1/R_2	$s^{1/3}$	$\Sigma^{1/3}$	$s^{1/3}$	$\Sigma^{1/4}$
t_1/t_2	$s^{1/3}$	$\Sigma^{1/3}$	$s^{1/6}$	$\Sigma^{1/8}$
p_1/p_2	1^*	1^*	$(s^{1/3})^\dagger$	$(\Sigma^{1/4})^\dagger$

$*p = \rho gk$

$\dagger p = \rho gk$ or $\rho g(d + k)$

6. When hydrostatic pressure, p, is interpreted as lithostatic pressure at the explosive burial depth, d, for both "mass-gravity" and "energy-gravity" scaling, three quantities, Y, ν, and c, contribute to violation of similarity in usual experiments. If these quantities could be scaled to insure similarity, they would have to be scaled in significantly different ways, depending on which scaling is accepted.

7. When hydrostatic pressure is taken to be both lithostatic and atmospheric, then for both "mass-gravity" and "energy-gravity" scaling, four terms arise to violate similarity in usual cratering experiments. If these quantities, Y, ν, c, and p, could be scaled to insure similarity, they would scale by very different amounts, depending on the scaling chosen. In particular, by "mass-gravity" scaling, atmospheric pressure is scaled by a cube-root rule, while for "energy-gravity" scaling, a fourth-root rule scales atmospheric pressure.

8. For "energy-gravity" scaling, explosive charge radius should be scaled by the fourth root of energy. In cratering experiments with the same type of chemical explosive, $a \propto E^{1/3}$; hence, similarity required by "energy-gravity" rules is violated in these experiments.

9. Moisture content and void ratio never violate similarity if they are constant for all experiments.

There is good experimental evidence to indicate that σ in the scaling rules of Equation 1 or 3 may be interpreted as peak shock pressure[22] or radial stress[23] associated with waves generated by explosions. Also in Equations 1 and 3, experiment[23] indicates that u, a, and τ may be regarded as particle velocity, particle acceleration, and period or time of arrival, respectively, of elastic waves from explosions. Thus, for example, it is found that at the same (cube-root) scaled distances and times, stresses and particle velocities are the same for two explosions, while accelerations are inversely proportional and periods directly proportional to the cube root of the charge weight.

In Equations 2 and 4, σ is probably not a radial stress of elastic waves or peak pressure of shock waves, since experiments have shown that elastic radial stresses obey the scaling rules given in Equations 1 and 3. Also, since u, a, and τ, interpreted as motions associated with initial waves, obey scaling rules of Equations 1 and 3, they likely do not have the same interpretation in Equations 2 and 4. While there is little experimental evidence to guide us in the interpretation of the velocity, u, and acceleration, a, in Equations 2 and 4, it is possible that these quantities represent the motions of particles which are ejected from those regions about explosives which ultimately constitute the crater void. For

example, u might be the ballistic velocity of a missile ejected[9,24] from the crater region or the velocity of an element of medium accelerated by the explosion gas bubble.[18] It would be scaled by rules in Equation 2 or 4 at a (scaled) distance, r, from the explosion and at a (scaled) time, t, which is long compared to the time the particle experiences motion from the shock or elastic wave.

Chapter 3

INFLUENCE OF SIMILARITY VIOLATIONS
ON SCALING OF CRATER DIMENSIONS

For any of the scaling rules derived (Equations 1 through 4), and from Table 1, it is seen that similarity is never achieved in actual cratering experiments, since there is always at least one variable (medium property, hydrostatic pressure, or charge radius) which should be scaled to insure similitude, but which remains constant in experiments.

In experiments, a constant viscosity, ν, violates similarity for all the scaling rules considered and, consequently, the scaling rules assumed to describe cratering will be in error to the extent that similarity has been violated. The effect of not being able to scale medium viscosity may be examined qualitatively. It seems plausible to assume that, for a given explosive in a given medium, crater dimensions will increase as viscosity is decreased. Thus, for example, r in Equation 1a must increase as the dimensionless quantity, to obtain $\nu/(\rho^2 c^3 W)^{1/3}$, decreases. When medium viscosity is constant, as for experiments, a decrease in the quantity $\nu/(\rho^2 c^3 W)^{1/3}$ also results from an increase in W, whereby a larger crater is produced. From this observation we may conclude that the viscous effects of a medium are more pronounced for smaller explosions, and model experiments with small explosions can be expected to produce smaller scaled (by Equation 1a) craters than would larger explosions. By the same argument, this conclusion pertains to all the scaling rules (Equations 1 through 4). <u>Thus, inability to scale medium viscosity results in larger scaled crater dimensions for larger explosions</u>.

For "mass-gravity" or "energy-gravity" scaling, the medium variables Y and c, in addition to ν, contribute to similarity violation when they remain constant in experiments. If it is assumed that crater dimensions will be increased when medium yield strength, Y, is decreased or when medium sonic velocity, c (ρc or ρc^2), is decreased, then it is seen from Equations 2a' and 4a' that scaled crater dimensions will increase as scaled yield strength and scaled sonic velocity are decreased. Since these scaled medium quantities also decrease for larger charges when Y and c are constant, the effects of Y and c on cratering are more inhibiting for smaller than for larger explosive charges. <u>Inability to scale any or all the medium properties (viscosity, yield strength, and sonic velocity) in cratering experiments will result in larger scaled crater dimensions for larger explosions when the scaling is by "mass-gravity" or "energy-gravity" rules</u>.

However, it may well be the case that in experiments with sufficiently large explosion masses or energies, similarity is nearly achieved, since the dimensionless quantities involving ν, Y, and c become small for very large explosions. This implies that the manner in which the scaled quantities containing ν, Y, and c occur in the functions G and I is such that the function has nearly an asymptotic value with respect to these quantities for the larger explosions. If this is correct, then the cube-root rule of "mass-gravity" scaling or the fourth-root rule of "energy-gravity" scaling will be good approximations for the larger explosions, provided similarity is otherwise achieved.

It must also be true that the dimensionless quantities containing ν, Y, and c can never be completely insignificant, no matter how large the explosion, since it is viscosity, yield strength, and sonic velocity together with density, moisture content, etc., that describe differences in media. For example, since ν, Y, c, and ρ are greater for rock media than for soil, Equations 1 through 4 show that crater dimensions in rock will be smaller than those in soil if experiments in each medium are identical except for these differences in ν, Y, c, and ρ.

The hydrostatic pressure term, p, in the "mass" and "energy" scaling Equations 1 and 3 is atmospheric pressure which, as we have seen, is not a source of similarity violation when p is constant in experiments. The quantity p in the "mass-gravity" and "energy-gravity" scaling Equations 2 and 4 does not give rise to a similarity violation when p is lithostatic pressure, ρgd. However, when p in Equations 2 and 4 is regarded as the sum of lithostatic and atmospheric pressure, or just atmospheric pressure, then it is apparent that atmospheric pressure must be scaled to preserve similitude among experiments. Keeping atmospheric pressure constant in experiments violates a similarity requirement, the effect of which is to obtain larger scaled crater dimensions for the larger explosions, assuming that increased atmospheric pressure results in a smaller crater from a given explosive in a given medium.

With "energy-gravity" scaling, charge radius is not scaled by the fourth-root rule as required from similarity, but scales by the cube-root rule in actual experiments where explosives of constant composition are used. This means that the explosive with $a \propto E^{1/3}$ is relatively more energetic than when $a \propto E^{1/4}$, as required. Consequently, the larger explosions are expected to produce scaled (fourth-root) crater dimensions which are larger than for smaller explosions. If $E_2 > E_1$ are the energies of two cratering explosions with the same explosive, then $a_2/a_1 = (E_2/E_1)^{1/3}$ in experiments; but, by "energy-gravity" scaling, it is necessary that $a_2/a_1 = (E_2/E_1)^{1/4}$, which is not possible if the same explosive is used. The effective energy release, $\mathscr{E}_2$, of the charge E_2 relative to E_1, which satisfies the similarity requirement on charge radius, is $\mathscr{E}_2 = (E_2^4/E_1)^{1/3}$. Crater dimensions should not be expected to scale by $\mathscr{E}^{1/4}$, however, since now the ratio of effective energy densities among explosions is not constant. The ratio of actual energy density, $E / \frac{4\pi}{3} a^3 \rho_x q$, to effective energy density, $\mathscr{E} / \frac{4}{3}\pi a^3 \rho_x q$, is proportional to $E^{-1/3}$ (ρ_x is density of explosive; q is specific energy release of explosive). As a result, the larger

explosions will have a smaller energy density relative to smaller explosions, and crater dimensions scaled by $\mathscr{E}$ should be smaller for the larger explosions than for the smaller explosions. <u>Thus, if fourth-root scaling is correct and if similar experiments are to be conducted, the same explosives cannot be used in experiments.</u> Different explosives must be used whose properties satisfy $\rho_{x1}q_1/\rho_{x2}q_2 = (E_1/E_2)^{1/4}$.

It is seen, then, that for the cube-root rules of "mass" or "energy" scaling, only one variable, medium viscosity, enters to destroy similitude in cratering experiments when it remains constant. For the cube-root rule of "mass-gravity" scaling and the fourth-root rule of "energy-gravity" scaling, four variables (medium viscosity, yield strength, sonic velocity, and atmospheric pressure) can give rise to similarity violations when they remain constant in experiments. In fourth-root scaling, a fifth variable, charge radius, can also be a source of similarity violation if the same explosive is used in experiments. <u>The qualitative effect of any or all these violations of similarity is to produce larger scaled crater dimensions for the larger explosions.</u>

The degree of influence of a similarity violation may be more marked for one crater dimension than for another; for example, since the functions I_1, I_2, and I_3 of Equation 4 are in general different from one another, the deviations from fourth-root scaling may be more apparent for radius than for crater depth or volume. Consequently, it could be possible to obtain the perplexing result that one crater dimension--radius, for example--would appear to scale better by the cube-root rule, while crater depth would scale best by the fourth-root rule. Such an observation has actually been made[25] for a particular set of crater data.

Four different scaling rules have been derived from dimensional analysis. Only one rule can be correct and the other three wrong. Suppose one were obliged to perform with different-sized explosives two cratering experiments whose results (e.g., crater dimensions) must scale. Suppose further that one were given the ability to vary or scale any or all the variables entering into the experiment, except sonic velocity which must be different for the two experiments, so that similitude can always be achieved. Also assume that measurements of infinite precision can be made. Before performing the two experiments, an experimenter would give serious thought to deciding which of the four scaling rules were correct since, if a wrong choice is made, the results of his experiment will not scale.

If the experimenter decides that gravity is not significant to cratering, he would choose the "mass" scaling rule or the "energy" scaling rule to design his experiment. He could, for example, keep ρ, Y, and p constant in both experiments and adjust the viscosities in the ratio $\nu_1/\nu_2 = (c_1/c_2)(W_1/W_2)^{1/3}$ or $(c_1/c_2)^{1/3}$ $(E_1/E_2)^{1/3}$, depending on whether he chooses "mass" or "energy" scaling. After learning of Ericsson and Edin's work,[21] he would probably choose "energy" scaling. If, however, the experimenter decides that gravity is important in cratering, he would choose either "mass-gravity" or "energy-gravity" scaling. For the former scaling,

he would adjust medium properties so that $\nu_1/\nu_2 = (W_1/W_2)^{1/2}$, $Y_1/Y_2 = (W_1/W_2)^{1/3}$, and $c_1/c_2 = (W_1/W_2)^{1/6}$, while for the latter he would have $\nu_1/\nu_2 = (E_1/E_2)^{3/8}$, $Y_1/Y_2 = (E_1/E_2)^{1/4}$, and $c_1/c_2 = (E_1/E_2)^{1/8}$. If he cannot establish whether or not atmospheric pressure is significant, he would, nevertheless, scale this pressure by the cube root or fourth root of explosive mass or energy just to play it safe. By accepting Ericsson and Edin's results as being applicable when gravity is important, he would probably decide on "energy-gravity" scaling. On the basis of Ericsson and Edin's experiments with blast waves in air, the experimenter would likely decide that the fundamental scaling rules are either the cube-root scaling of Sachs or the fourth-root scaling of Haskell.

Before making a final commitment, our harried experimenter undoubtedly would make a literature search to see what experiments had been performed and what light they might shed in assisting him to make his final choice.

From a multitude of crater data[1,3] from small (less than 100 pounds TNT) chemical explosions, it is found that cube-root and not fourth-root scaling best describes the data. The fourth-root rule is definitely inadequate for scaling crater dimensions of small charges, implying that gravity is not significant to cratering. On the other hand, as will be seen in the next chapter, a large set of consistent data from carefully executed experiments in Nevada soil with large chemical explosions (10^2 to 10^6 pounds TNT) shows unquestionably that cube-root rules do not scale crater dimensions from these explosions. At the same time, crater dimensions from these experiments cannot be exactly scaled by fourth-root rules either; however, it is found that they are fairly well scaled by an empirical 3/10 rule. The Nevada cratering experiments suggest strongly, then, that similarity among experiments is not achieved since, otherwise, either cube-root or fourth-root rules, whichever is fundamental to cratering phenomena, would scale the data. If similarity is grossly violated in experiments, then application of scaling rules to crater data is questionable and of limited value. In order to estimate crater dimensions for large explosions from small or model explosion data, with scaling rules which are not strictly valid, it will be necessary to delineate the sources of similarity violation, assess their relative influence, and attempt to account quantitatively for the deviations they induce in the scaling rules. But first it will be required to deduce which scaling rule, cube root or fourth root, is being violated.

Our experimenter under obligation would thus have found results of his literature search most disappointing. In addition to not finding any direct evidence to favor either cube-root or fourth-root scaling, he finds the Nevada cratering data which apparently deny both cube-root and fourth-root rules, and a further question not previously conceived--the reason for the empirical 3/10-scaling rule--has arisen.

A possible explanation of the Nevada crater scaling dilemma is found in Equation 4a'. For example, note that the dimensionless scaled crater radius $\pi_1 = r(\rho g/E)^{1/4}$ may be expressed in the equivalent form $\pi_1' = \pi_1^3 \pi_2 = pr^3/E$, where

$\pi_2 = p/(\rho^3 g^3 E)^{1/4}$. As we have seen, when p is lithostatic pressure there is no similarity violation arising from the term p. However, when p is lithostatic (ρgd) plus atmospheric (ρgk) pressure, and atmospheric pressure is not scaled in experiments, a violation of similitude occurs. The ratio of crater radii from two similar explosions is

$$\frac{r_1}{r_2} = \left[\left(\frac{\rho_2 g_2}{\rho_1 g_1}\right)\left(\frac{d_2 + k_2}{d_1 + k_1}\right)\right]^{1/3}\left(\frac{E_1}{E_2}\right)^{1/3} = \left(\frac{\rho_2 g_2}{\rho_1 g_1}\right)^{1/4}\left(\frac{E_1}{E_2}\right)^{1/4} \tag{9}$$

When atmospheric pressure is not scaled and $k_1 = k_2$, then Equation 9 is not absolutely correct. Since k is of the order of 20 feet of earth material and since a large majority of cratering experiments have been conducted at burial depths of this magnitude or less, it is possible that inability to scale crater dimensions over a wide range of explosion energies may be a result of similarity violation produced by unscaled atmospheric pressure. As has been noted, the effect of not scaling atmospheric pressure is to produce larger scaled craters for larger explosions than for smaller explosions; or, stated in another way, for large explosions at their deeper burial depths, the restraining effect of atmospheric pressures is less, relative to lithostatic pressure, than for small explosions, and larger scaled crater dimensions are obtained.

In Equation 9, referred to henceforth as the approximate "overburden" scaling rule, it is seen that for small explosions where the burial depths and, hence, lithostatic pressures are much less than atmospheric, the depths, d_1 and d_2, for two small explosions may be neglected in comparison to the constant value of k, and the ratio of crater dimensions will be equal to the cube root of the ratio of explosion energies. For very large explosions where lithostatic pressure is much greater than atmospheric, k may be neglected in comparison with d and, by the similarity condition, $r_1/r_2 = d_1/d_2$, fourth-root scaling results. Thus, the approximate "overburden" scaling relation of Equation 9, in which k is constant, reduces to cube-root scaling for small explosions and renders fourth-root scaling for large explosions. For explosions of intermediate size, it is reasonable to expect from Equation 9 a scaling such as 3/10, which is between cube root and fourth root. This description allows us to explain at least qualitatively a number of experimental results, and it suggests some cratering experiments which have not yet been performed.

From Equation 9 we may make the following observations regarding the influence of atmospheric pressure (assuming similitude is otherwise achieved):

1. In the limit of very small explosive charges where burial depth which produces a crater is much less than k, cube-root scaling results: $r \propto E^{1/3}$ for $k \gg d$ and as $E \to 0$.

2. For large explosions at cratering depths where lithostatic pressure is much greater than atmospheric pressure, fourth-root scaling prevails: $r \propto E^{1/4}$ for $d \gg k$ as $E \rightarrow$ large.

3. For a certain range of explosion energies, E, when $d \approx k$, $r \propto E^m$, where $1/4 < m < 1/3$.

4. For a given explosion energy, burial depth, and medium, the crater dimension can be changed by varying atmospheric pressure, k.

5. In vacuo, $k = 0$, and fourth-root scaling results for buried explosions of any energy.

Using different arguments with gravitational acceleration as a significant variable, Pokrovskii and Fedorov[26] arrived at conclusions similar to those noted in items 1, 2, and 3 above.

It is seen that one way of testing the validity of Equation 9 is to perform cratering experiments in which g can be varied. Likewise, the validity of Equation 9 could be tested by varying k in experiments.

From the "mass-gravity" rule of Equation 2a', a relationship analogous to Equation 9 is obtained,

$$\frac{r_1}{r_2} = (\rho_2/\rho_1)^{4/9} \left[(d_2 + k_2)/(d_1 + k_1) \right]^{1/3} (W_1/W_2)^{4/9},$$

from which for small explosions where $d \ll k$, $r \propto W^{4/9}$, and for large explosions $r \propto W^{1/3}$ when $d \gg k$. Since data from small charge experiments reveal that $r \propto W^{1/3}$ rather than $W^{4/9}$, the "mass-gravity" scaling rule can probably be rejected on this basis. Our puzzled experimenter will have some small degree of confidence restored by this observation, since it lends some support to his earlier assumption that the Ericsson-Edin work indicates "energy-gravity" scaling to be preferable to "mass-gravity" scaling.

Chapter 4

COMPARISON OF SCALING RULES WITH
EXPERIMENTAL DATA

Crater data in Table 2 from explosions in Nevada desert alluvium have been scaled by each of the rules, Equations 1 through 4, obtained from dimensional analysis. Crater radii, depths, and volumes scaled by cube-root rules are shown in Figures 1a, 1b, and 1c, by the fourth-root rule in Figures 2a, 2b, and 2c, by the empirical 3/10 rule in Figures 3a, 3b, and 3c, and also by the approximate "overburden" scaling rule of Equation 9 in Figures 4a, 4b, and 4c. Symbol notation of Figure 1a is retained throughout. For comparison with nuclear explosion data, a least-squares-fit line (see Table 3) for the large ($W \geq 20$ tons) chemical explosion data is shown in each illustration.

In Figures 1a, 1b, and 1c, it should be noted that the spread in data is great and that the data are very poorly scaled by cube-root rules. Of particular significance is the observation that scaled crater dimensions from the larger explosions are systematically less than those of smaller explosions, also that the much larger nuclear explosions have scaled dimensions which are considerably less than those of the chemical explosions.

First attempts to account for the large difference in (cube-root) scaled dimensions between nuclear and chemical explosions resulted in definitions of nuclear-to-chemical explosive "efficiencies."[27,28,29] These efficiency factors are variable, depending on the crater dimension considered and on the scaled burst depth, and are not accurately known, owing to scatter in chemical explosion data and to sparseness of nuclear explosion data. In general, introduction of efficiency factors has not improved our ability to predict cratering from nuclear explosions with the use of available data from chemical explosives.

As noted in Chapter 1, cube-root rules result from Lampson "mass" scaling, from Sachs "energy" scaling, and from "mass-gravity" scaling. In "mass" and "energy" scaling, only medium viscosity, which should be scaled but is kept constant in experiments, violates the similarity requirement. In "mass-gravity" scaling, constancy of any or all of the variables ν, Y, c, and k in experiments violates similarity (see Table 1), and the qualitative effect of each of these similitude transgressions is to produce larger scaled crater dimensions for the larger explosions.

TABLE 2

Apparent Crater Dimensions from Explosions
in Desert Alluvium

Shot		DOB	Radius (ft)	Depth (ft)	Volume (ft^3)	Charge weight* (lb)
Mole	202[†]	6.35	11.40	5.90	1,027	256
	203[†]	3.17	8.35	4.10	358	256
	204[†]	1.65	9.35	2.40	364	256
	205[†]	0.83	8.90	2.20	312	256
	206	0.00	6.35	1.70	129	256
	207	-0.83	4.05	1.40	37	256
	212[†]	6.35	11.20	6.07	1,174	256
	401[†]	3.17	10.59	5.45	837	256
	402[†]	4.76	11.05	6.25	961	256
	403[†]	0.83	8.36	3.37	301	256
	404[†]	6.35	12.10	6.12	1,195	256
	405[†]	1.65	9.24	4.60	511	256
	406[†]	3.17	9.95	4.22	686	256
Jangle	HE 1[†]	2.05	18.50	6.70	2,010	2,560
	HE 2[†]	5.13	39.00	15.00	35,000	40,000
	HE 3[†]	6.84	20.27	10.80	6,000	2,560
	HE 4	-2.05	6.90	1.90	110	2,560
	HE 5[†]	4.10	19.40	7.50	4,000	2,560
	HE 6[†]	3.01	19.80	6.10	3,600	2,560
	HE 7[†]	2.60	19.00	6.70	3,300	2,560
	HE 9[†]	0.84	8.30	3.50	270	216
	HE 10[†]	3.00	11.30	5.50	860	216
Sandia, Series I	2[†]	9.53	15.12	7.86	2,146	256
	4	15.90	11.32	1.77	368	256
	8[†]	6.35	13.13	7.30	1,489	256
	9[†]	9.53	14.14	7.16	1,930	256
	10	12.70	13.40	4.10	1,093	256
	11	15.90	6.53	0.38	236	256
	12	19.05	9.36	2.30	256	256
	15	25.40	4.18	0.45	31	256
	16	12.70	14.19	6.70	2,220	256
	17	19.05	5.68	1.70	55	256

*Pounds of TNT or TNT equivalent, 1 kiloton = 4.19 x 10^{19} ergs.
[†]Data used in regression analysis discussed in the text.

TABLE 2
(Cont)

Shot		DOB	Radius (ft)	Depth (ft)	Volume (ft³)	Charge weight* (lb)
Sandia, Series II	1	29.80	31.00	-0.63	-584	256
	2	28.50	37.70	-0.83	-1,079	256
	3	26.10	32.30	-1.03	-1,187	256
	4	25.50	2.35	1.15	16.4	256
	5	23.30	3.03	0.30	17.9	256
	6	22.60	4.39	1.00	170	256
	7	19.70	8.13	1.01	121	256
	8	19.00	10.07	1.60	297	256
	9	16.40	14.29	2.61	716	256
	10	16.10	14.10	4.55	1,077	256
	11	13.10	14.69	5.43	1,670	256
	S-12	0.00	8.57	2.49	161	256
	S-13	0.00	8.34	2.60	267	256
Stagecoach	I	80.0	57.0	7.9	49,000	40,000
	II†	17.1	50.5	23.6	83,650	40,000
	III†	34.2	58.6	29.2	144,600	40,000
Scooter†		125	154	74.5	2.642×10^6	1×10^6
5000 LB		0.00	18.50	4.75	2,950	5,000
3000LB†		14.31	26.2	13.9	12,000	3,000
Jangle	S	-3.48	45.0	19.0	49,270	2.4×10^6
	U†	17.00	130.0	53.0	9.73×10^5	2.4×10^6
Teapot	S†	67.0	146	90.0	2.6×10^6	2.4×10^6
Sedan†		635	611	323	1.79×10^8	2×10^8

*Pounds of TNT or TNT equivalent, 1 kiloton = 4.19×10^{19} ergs.
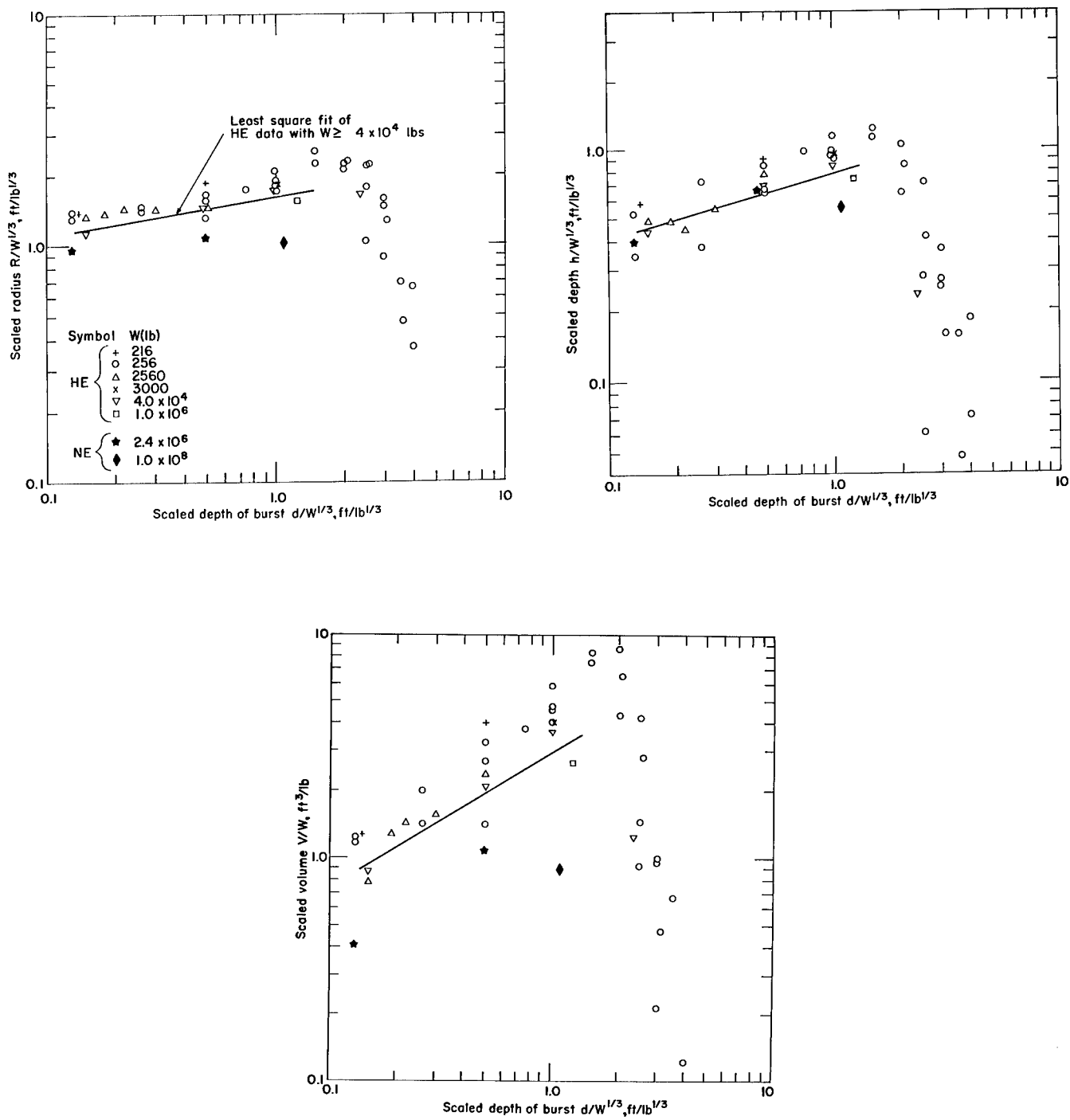†Data used in regression analysis discussed in the text.

Figure 1 Apparent crater dimensions from buried explosions
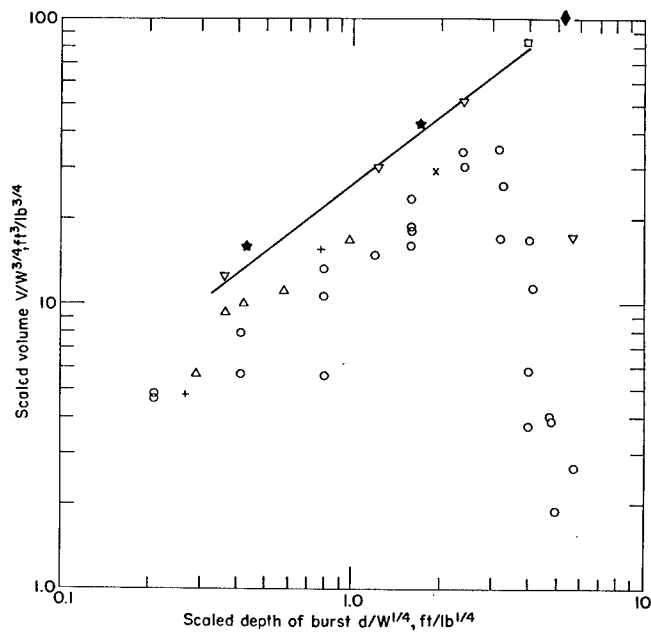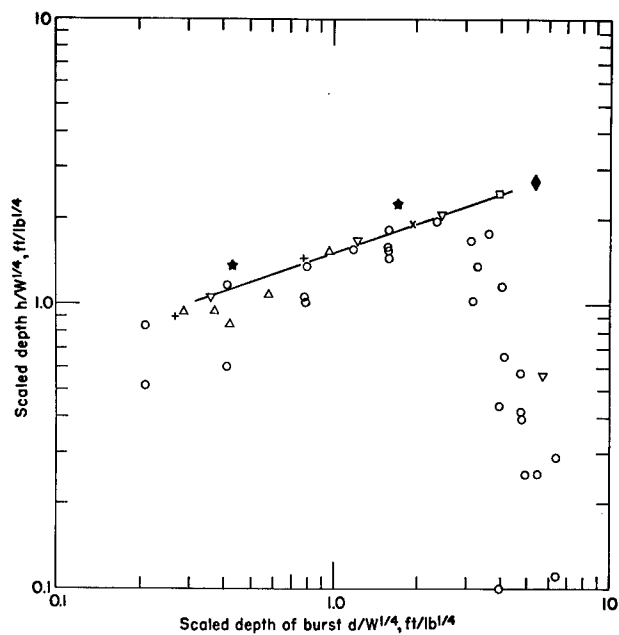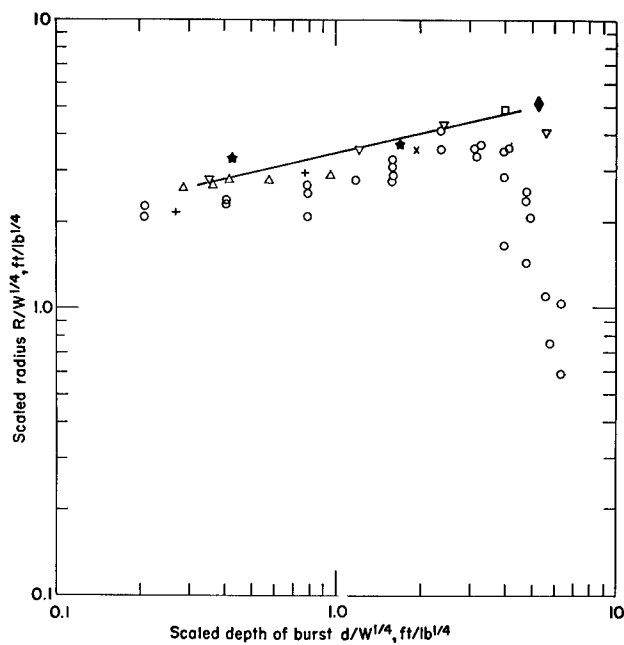in alluvial soil scaled by cube-root rule.

Figure 2 Apparent crater dimensions from buried explosions
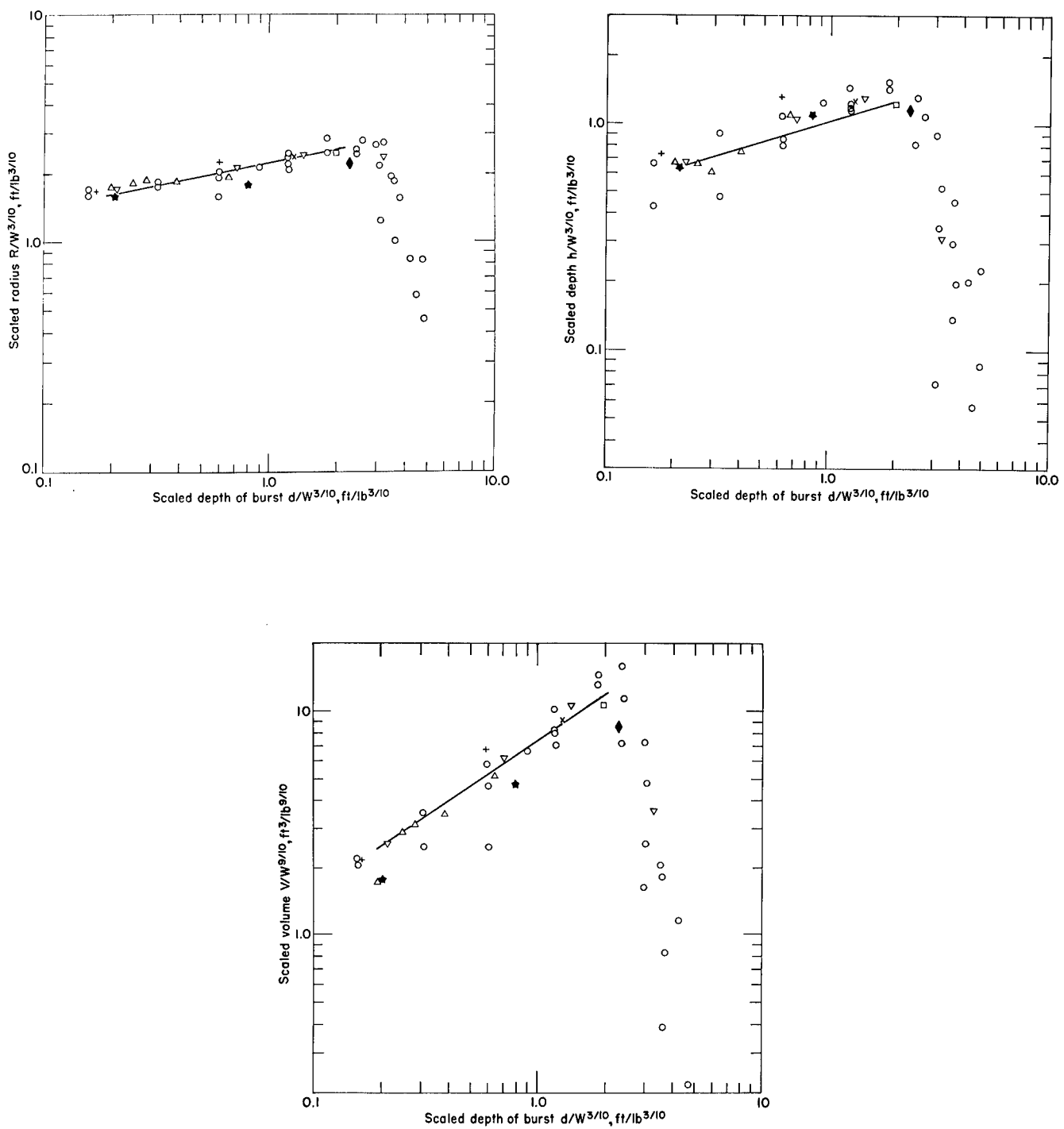in alluvial soil scaled by fourth-root rule.

Figure 3   Apparent crater dimensions from buried explosions
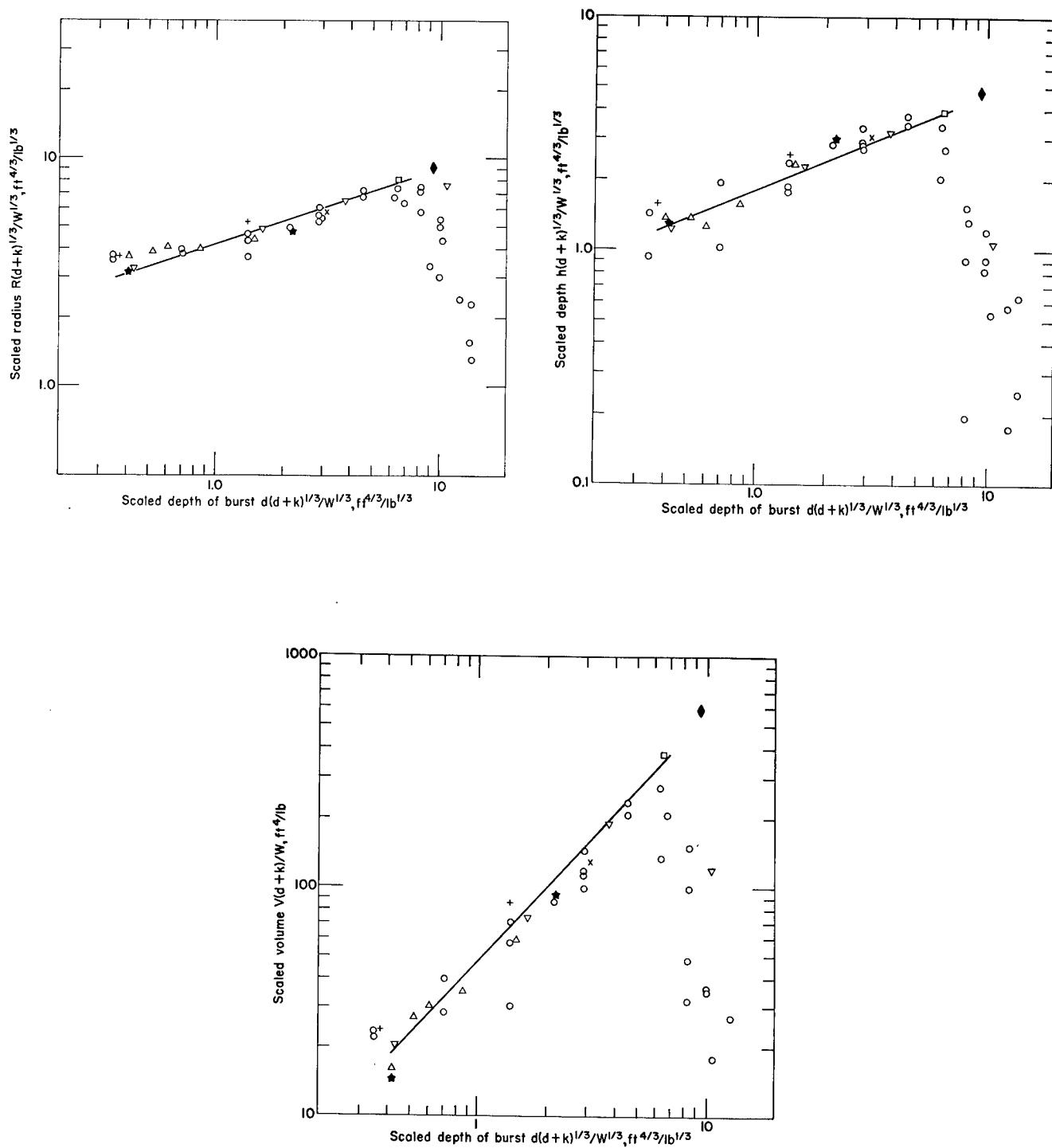in alluvial soil scaled by empirical 3/10 rule.

Figure 4 Apparent crater dimensions from buried explosions in alluvial soil scaled by approximate overburden scaling rule (k = 18.3 feet).

TABLE 3

Least-Squares-Fit Values and Their Deviations for Desert
Alluvium Crater Data Scaled by Various Rules

Data	Dimension	1/3				3/10				1/4				Overburden			
		A	σ_A	m	σ_m	A	σ_A	m	σ_m	A	σ_A	m	σ_m	A	σ_A	m	σ_m
All HE and NE n = 29	r	0.241	0.019	0.167	0.038	0.336	0.009	0.165	0.021	0.483	0.011	0.219	0.027	0.637	0.008	0.265	0.019
	h	-0.040	0.022	0.361	0.045	0.032	0.016	0.358	0.036	0.145	0.014	0.398	0.036	0.264	0.013	0.431	0.029
	V	0.577	0.051	0.710	0.103	0.838	0.023	0.705	0.055	1.241	0.028	0.853	0.073	1.665	0.021	0.980	0.048
HE only n = 26	r	0.263	0.012	0.177	0.024	0.347	0.008	0.181	0.019	0.473	0.010	0.209	0.028	0.640	0.008	0.250	0.020
	h	-0.022	0.021	0.381	0.042	0.040	0.016	0.381	0.039	0.135	0.014	0.397	0.038	0.262	0.014	0.430	0.035
	V	0.635	0.033	0.733	0.067	0.865	0.021	0.741	0.049	1.214	0.028	0.817	0.074	1.673	0.021	0.931	0.052
NE only n = 3	r	0.030	0.020	0.040	0.036	0.263	0.003	0.095	0.008	0.565	0.028	0.173	0.057	0.610	0.035	0.333	0.055
	h	-0.207	0.091	0.182	0.166	-0.009	0.053	0.223	0.120	0.252	0.033	0.283	0.067	0.290	0.030	0.421	0.047
	V	0.023	0.137	0.409	0.250	0.651	0.053	0.552	0.122	1.471	0.012	0.753	0.024	1.592	0.027	1.188	0.043
All HE and NE W ≥ 20T n = 7	r	0.142	0.042	0.139	0.086	0.311	0.019	0.153	0.046	0.550	0.011	0.204	0.025	0.619	0.012	0.331	0.020
	h	-0.142	0.037	0.253	0.075	0.005	0.019	0.267	0.047	0.211	0.019	0.313	0.041	0.272	0.013	0.420	0.022
	V	0.307	0.120	0.600	0.245	0.779	0.052	0.636	0.127	1.445	0.013	0.773	0.028	1.638	0.025	1.131	0.043
HE only W ≥ 20T n = 4	r	0.205	0.021	0.168	0.048	0.345	0.008	0.193	0.020	0.538	0.006	0.232	0.014	0.627	0.008	0.328	0.015
	h	-0.105	0.031	0.286	0.070	0.014	0.018	0.305	0.046	0.181	0.007	0.337	0.017	0.258	0.006	0.419	0.011
	V	0.466	0.063	0.605	0.143	0.868	0.025	0.676	0.065	1.426	0.015	0.788	0.037	1.680	0.024	1.065	0.045
HE only W ≤ 20T n = 22	r	0.277	0.012	0.187	0.023	0.347	0.010	0.179	0.023	0.456	0.010	0.172	0.027	0.640	0.009	0.225	0.023
	h	-0.002	0.023	0.408	0.046	0.049	0.020	0.402	0.045	0.129	0.017	0.395	0.046	0.263	0.016	0.436	0.043
	V	0.675	0.032	0.778	0.064	0.867	0.026	0.753	0.059	1.165	0.024	0.731	0.066	1.669	0.023	0.881	0.061

None of these sources of similarity violation can be invoked to explain the deviations from cube-root scaling observed in the data of Figures 1a, 1b, and 1c, which implies that cube-root rules may not be the fundamental scaling descriptions of crater dimensions.

In Figures 2a, 2b, and 2c, desert alluvium crater data are scaled by the fourth root of the explosion energy. Scaled in this manner, spread in data is still great. However, it is seen that now the larger explosions have generally greater scaled dimensions. Thus, it is possible that any or all of the variables ν, Y, c, k, and a account for the lack of fourth-root scaling because of similarity violation, since any one of these variables, when not scaled in experiments, has the effect of producing larger scaled crater sizes for the larger explosions. The implication of this observation is that the fourth-root rule of "energy-gravity" scaling may be the fundamental scaling description of crater dimensions.

An attempt to correct for the similarity violation produced by the nonscaling charge radius is illustrated in Figures 5a, 5b, and 5c. Crater dimensions have been scaled by the effective charge energy, $\mathscr{E}$, so that the similarity requirement, $a_1/a_2 = \left(\mathscr{E}_1/\mathscr{E}_2\right)^{1/4}$, is met. Effective charge energy was determined relative to the 256-pound charges with $\mathscr{E} = \left(E^4/E_1\right)^{1/3}$, $E_1 = 256$, and E is actual charge energy. It is seen that, with this type of correction, the smaller charges produce larger scaled crater dimensions and that a uniform scaling does not result. This corrected scaling is just cube-root scaling, except for the correction factor $(256)^{1/12}$ applied to a scaled linear dimension (compare with Figures 1a, 1b, and 1c). The reason data are not scaled by this correction is, as mentioned above, that the experiments are still not similar because the effective energy density of charges is not constant. Smaller scaled crater dimensions for larger charges occur because the ratio of actual energy density to effective energy density is less for the larger charges. Figures 5a, 5b, and 5c point out the magnitude of deviations from a scaling rule which may be expected from sources of similarity violation. Also it appears that, if fourth-root scaling is correct, a major contribution to nonsimilarity is the charge radius when experiments are performed with constant-composition explosives. An interpretation of the scatter in Figures 1a, 1b, and 1c is possible with Figures 5a, 5b, and 5c; i.e., data scatter widely when scaled by cube-root rules, probably because cube-root scaling is incorrect and because they should be scaled by fourth-root rules. Data are not precisely scaled by fourth-root rules, however, primarily because charge radii in experiments are not fourth-root scaled as required. The relative deviations in scaled dimensions from one charge size to another, witnessed in Figures 1a, 1b, and 1c, are accounted for precisely by the arguments which produced Figures 5a, 5b, and 5c.

In Figures 3a, 3b, and 3c, alluvium crater data have been scaled by the empirical 3/10 rule. It is seen that the chemical explosive data are nicely scaled by this rule over the range of energies, 200 pounds to 500 tons. Data from the three
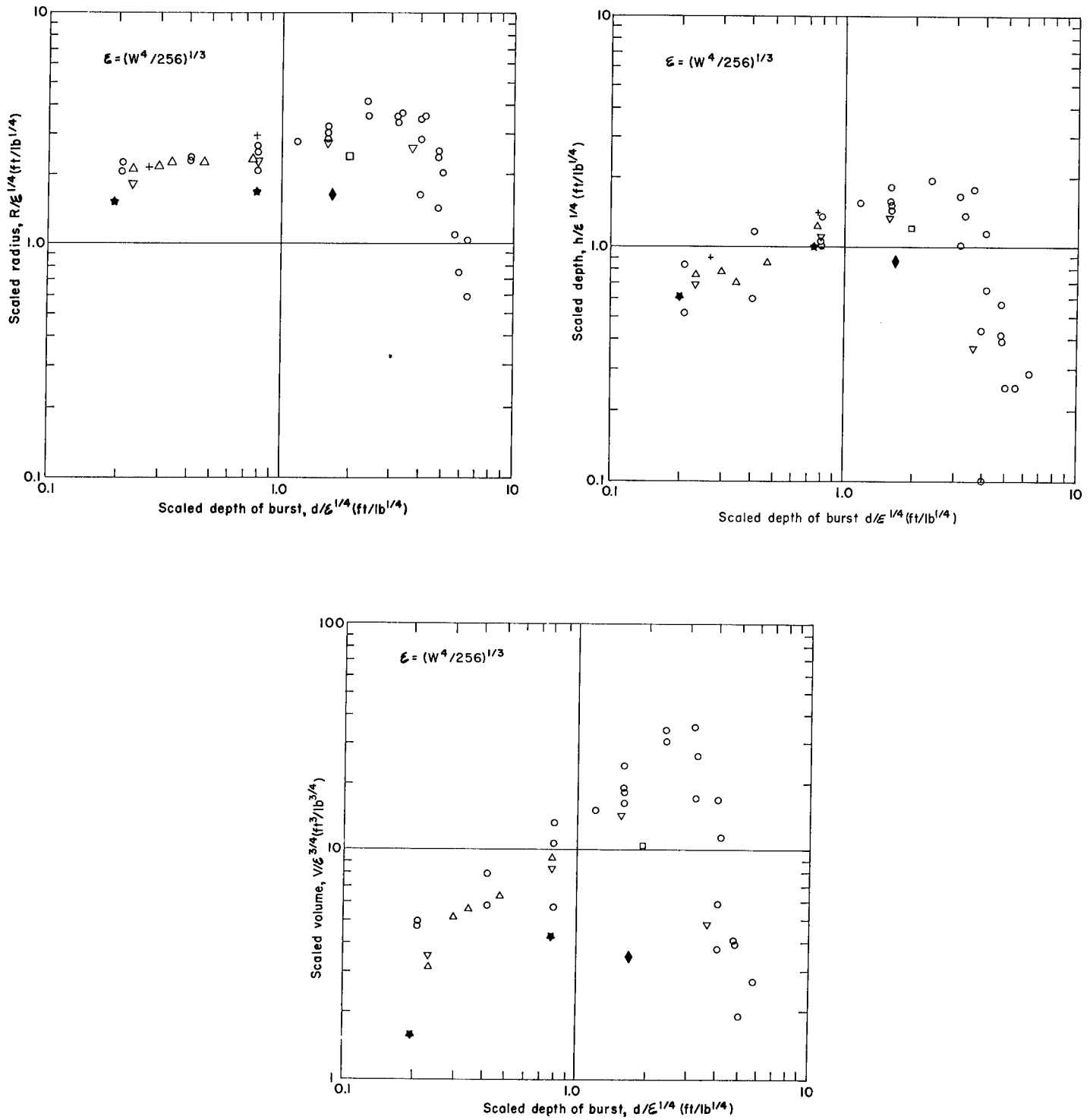
Figure 5 Apparent crater dimensions from buried explosions in alluvial
soil scaled by fourth-root rule using effective yield
$\mathscr{E} = \left(E^4/E_1\right)^{1/3}$ relative to E_1 = 256 pounds.

nuclear explosions do not scale well with the chemical explosive data, which leads one to the conclusion that, if 3/10 scaling is somehow a correct description of cratering, then chemical and nuclear explosions are not equally effective in producing craters. As a result, in order to utilize data from only chemical charges in other media for predicting nuclear crater dimensions, some sort of efficiency factors will be required. Such a procedure greatly reduces confidence in crater-size prediction capability, particularly when megaton energies are considered.

We have seen that the desert-alluvium crater data are not exactly scaled by either cube-root or fourth-root rules, but, with the exception of nuclear crater data, they are well scaled by the empirical 3/10 rule. A possible explanation of these results was given by Equation 9 where, as we saw in Chapter 3 (item 3), for a certain range of explosion energies, some scaling, intermediate to cube root and fourth root, prevails if the quantity p is interpreted as atmospheric or total hydrostatic pressure at the bural depth of an explosion. It may be the case that 3/10 scaling is this intermediate scaling rule which results primarily from our inability to scale atmospheric pressure to meet the similarity requirement. If this is true, then Equation 9 may be a reasonably good approximation for a scaling rule over that range of explosion energies where the effect of not being able to scale k is most significant. To test this possibility, crater data have been scaled by the approximate "overburden" rule in Figures 4a, 4b, and 4c with the use of k = 18.3 feet.

One means of attempting to distinguish which, if any, of the scaling rules best describes the data is to perform a regression analysis and obtain least-square fits of the data to some assumed function, compute some correlation coefficient on a comparable basis, and judge a best rule as that which renders maximum correlation. From Figures 1 through 4, it is seen that simple and reasonable functions to assume are the linear relationships

$$\Lambda_x^c = A + m\Lambda_d \tag{10}$$

where $\Lambda_x = \log \lambda_x$, $\Lambda_d = \log \lambda_d$, λ_x is a scaled crater dimension, radius, depth, or volume, λ_d = scaled depth of burial, and Equation 10 is restricted to a definite range of values in Λ_D. Equation 10 is an assumed and extremely simplified form of the functions in Equations 1 through 4. Standard deviation is computed by the formula[30]

$$\sigma^2 = \sum_{i=1}^{n} \left(\Lambda_{xi}^c - \Lambda_{xi}^o \right)^2 \Big/ (n - 2)$$

where Λ_x^c is the logarithm of a scaled crater dimension, calculated by Equation 10 whose constants A and m are determined from a least-squares fit, Λ_{xi}^o are the observed values, and n is the number of data. A measure of the uncertainty in the values of A and m is obtained from the formulas

$$\sigma_m^2 = n\sigma^2 \Bigg/ \left[n \sum_{i=1}^{n} \Lambda_{di}^2 - \left(\sum_{i=1}^{n} \Lambda_{di} \right)^2 \right]$$

and

$$\sigma_A^2 = \sigma^2 \sum_{i=1}^{n} \Lambda_{di}^2 \Bigg/ \left[n \sum_{i=1}^{n} \Lambda_{di}^2 - \left(\sum_{i=1}^{n} \Lambda_{di} \right)^2 \right]$$

where Λ_{di} represents scaled burial depth for the ith experiment. Only those data marked with a dagger in Table 2 were utilized in the regression analysis. These data determine the cratering curve over which crater dimensions increase with increasing burial depth. Data which delineate the monotonically decreasing sections of the cratering curves were not treated because data in this region from different-sized charges are not sufficiently extensive (see Table 2).

Results of the regression analysis are shown in Table 3 for the various scaling rules and for several groupings of the data. It is emphasized that the constants A and m tabulated in Table 3 apply only to the range of scaled depths covered by the data. They do not apply to craters from surface-burst explosions nor to craters from explosions at scaled depths greater than those treated. (Least-squares-fit lines in Figures 1 through 4 are those obtained only from HE data with $W \geq 20$ tons, using constants of the last row of Table 2.)

A measure of how well the data fit the regression line is given by ϕ^2, called the coefficient of determination[30] and defined by

$$\phi^2 = \sum_{i=1}^{n} \left(\Lambda_{xi}^c - \bar{\Lambda} \right)^2 \Bigg/ \sum_{i=1}^{n} \left(\Lambda_{xi}^o - \bar{\Lambda} \right)^2 ,$$

in which

$$\bar{\Lambda} = (1/n) \sum_{i=1}^{n} \Lambda_{xi}^o \text{ and } 0 < \phi^2 < 1 .$$

If all data points lie close to (on) the line, then ϕ^2 will be close to (actually) one, but as scatter of the data becomes greater, ϕ^2 will become smaller. In our application of fitting crater data to a regression line with different scaling rules, ϕ^2 represents a measure of the authenticity of one scaling rule compared to another. Values of ϕ^2 for the different scaling rules and for several groupings of the data are shown in Table 4.

TABLE 4

Coefficients of Determination, ϕ^2, from Desert Alluvium Crater Data Scaled by Four Rules and for Several Data Groupings

Scaling rule	Dimension	$2 \times 10^2 < W \leq 2 \times 10^8$ All HE and NE n = 29	$2 \times 10^2 < W \leq 10^6$ Only HE n = 26	$2 \times 10^6 < W \leq 2 \times 10^8$ Only NE n = 3	$4 \times 10^4 \leq W \leq 2 \times 10^8$ HE and NE with W ≥ 20T n = 7	$4 \times 10^4 \leq W \leq 10^6$ Only HE with W ≥ 20T n = 4	$2 \times 10^2 < W \leq 3 \times 10^3$ Only HE with W < 20T n = 22
1/3	r	0.4180 (0.6038)	0.6868	0.5478	0.3437 (0.7091)	0.8619	0.7653†
	d	0.7053 (0.7890)	0.7704	0.5452	0.6962 (0.9324)	0.8923	0.8002†
	v	0.6373 (0.7848)	0.8347	0.7280	0.5450 (0.8332)	0.8997	0.8801
3/10	r	0.7004 (0.7658)	0.7926†	0.9932*	0.6895 (0.8543)	0.9778	0.7541
	d	0.7814 (0.8095)	0.8027	0.7745	0.8657 (0.9490)	0.9558	0.7961
	v	0.8607† (0.9001)	0.9036†	0.9536	0.8343 (0.9427)	0.9817	0.8896†
1/4	r	0.7043† (0.6522)	0.7007	0.9012	0.9307† (0.9020)	0.9927†	0.6675
	d	0.8155† (0.7904)	0.8208†	0.9464†	0.9208† (0.8963)	0.9949†	0.7843
	v	0.8363 (0.8079)	0.8370	0.9990*	0.9932* (0.9913)	0.9957†	0.8596
Over-burden	r	0.8826* (0.8668)	0.8653*	0.9736†	0.9814* (0.9750)	0.9956*	0.8317*
	d	0.8879* (0.8721)	0.8661*	0.9878*	0.9867* (0.9818)	0.9986*	0.8344*
	v	0.9395* (0.9321)	0.9306*	0.9987*	0.9929† (0.9904)	0.9965*	0.9136*

*Maximum value of ϕ^2 for a crater dimension within a data group.

†Value of ϕ^2 next greatest in magnitude for a crater dimension within a data group.

Data from only chemical (HE) and only nuclear (NE) explosions were treated separately to see whether or not ϕ^2 values would reveal anything about the relative effectiveness of nuclear and chemical explosions. Data from explosions whose energy release was greater than or equal to 20 tons of TNT were also treated separately to test arguments of the previous section, that similarity in fourth-root scaled experiments is more nearly achieved for the deeper and larger explosions. Also, data from explosions having a yield less than 20 tons were treated as a group to test the argument that, for these experiments, violations of similarity are greater, tending to produce greater deviations from fourth-root scaling and better agreement with cube-root scaling.

Examining each crater dimension in Table 4 within a data group (column), an asterisk is placed after that value of ϕ^2 which is maximum for the dimension considered. This indicates that the scaling rule (row) on which the asterisk falls best describes the data. A dagger is placed after that ϕ^2 value which is next greatest in magnitude.

Excluding column 6, it is seen that ϕ^2 values for any dimension and for all groupings of data are least for the cube-root rule, implying that this scaling is least capable of describing the data.

Values of ϕ^2 in column 6 test the hypothesis that, for smaller explosions, cube-root scaling tends to be more correct than fourth-root scaling because of non-scaling of atmospheric pressure. While "overburden" scaling still gives the best fit, it is seen (comparing columns 2 and 6) that, for radius and depth, ϕ^2 for 1/3 scaling has increased and ϕ^2 for 1/4 scaling has decreased. For volume, ϕ^2 has increased for both cube-root and fourth-root scaling, but the increase in ϕ^2 is greatest for cube-root scaling.

Considering all or only HE data (columns 1 and 2), one finds that the approximate "overburden" scaling rules describe the data best, with fourth-root or 3/10 scaling being the next best over-all rule.

In column 3, where only NE data are treated, it is seen that the 3/10 rule is the best scaling description for crater radius, but "overburden" or fourth-root scaling is to be preferred for depth and volume over the range of yields, 1 to 100 kilotons, covered by the three data points.

Comparing ϕ^2 values for those explosions whose energy release was greater than or equal to 20 tons (column 4), it is seen that fourth-root or "overburden" scaling is better than cube-root and 3/10 scaling. This observation, derived from the ϕ^2 value criterion, is interpreted as an indication that the fourth-root rule is the fundamental rule for scaling crater dimensions and that the "overburden" rule is approximately correct in accounting for deviations from fourth-root scaling when similarity among cratering experiments is not achieved.

In columns 1 and 4, the values of ϕ^2 in parentheses are those with the Sedan 100-kiloton data excluded. Data from the Sedan experiment produce a definite decrease in ϕ^2 for cube-root and 3/10 scaling, whereas ϕ^2 for fourth-root scaling remain about the same or increase slightly. This is felt to be another indirect evidence that fourth-root scaling is a better description of cratering than is cube-root or the empirical 3/10, rule, particularly for the larger explosions.

Comparing ϕ^2 values in columns 1 and 2 or those in columns 4 and 5 for a given crater dimension, one finds that, when both HE and NE data are considered, ϕ^2 is usually greater when only HE data are treated. This may possibly be an implication that some difference exists between chemical and nuclear explosions. Relative effectiveness of nuclear explosions in producing craters compared to chemical explosions cannot be meaningfully established from available data because of the uncertainty associated with scaling rules. It is not even clear whether nuclear explosions are more effective or less effective than chemical explosions. This is readily apparent from comparison of nuclear and chemical explosion data in Figures 1 through 4. If cube-root scaling is accepted, then nuclear explosions are considerably less effective than chemical explosions (Figures 1a, 1b, and 1c). With fourth-root scaling, it is seen (Figures 2a, 2b, and 2c) that nuclear explosions are more effective. By 3/10 scaling, nuclear explosions are less effective than chemical explosions for radius and volume dimensions, and greater or about equal to chemical explosions for the crater depth dimension. The question of relative effectiveness cannot be resolved until one of the scaling rules is definitely established as being best, or until two identical experiments, one with nuclear and one with chemical explosives, are performed.

On the basis of the ϕ^2 criterion and from observation of the number of asterisks and daggers associated with a scaling rule in Table 4, it is apparent that, over a wide range of yields, overburden scaling is the best rule. Fourth-root scaling is the next best rule for scaling crater dimensions, particularly for large yields. Cube-root scaling is the least likely valid rule for the range of yields considered, and 3/10 scaling appears to be the next poorest rule.

Other evidence bearing on the question of scaling is found in the experiments of Viktorov and Stepenov,[31] who showed directly the influence of gravity in explosion-produced craters. In this work, identical experiments were performed in accelerated frames, where the accelerations were 1, 25, 45, and 66 g's. As acceleration was increased, it was found that crater dimensions were greatly reduced, as might have been expected only from the fourth-root scaling rule, e.g., Equation 4a' or 8. Shown in Figure 6 is a reproduction of Figure 6 in Viktorov and Stepenov's paper. Solid lines represent results of experiments for three different-size charges at a given depth of burial; the dashed line shows the functional dependence of acceleration on crater dimension--in this case, $V \propto g^{-3/4}$, according to the fourth-root rule. The position of the dashed line in the figure is not significant; only its curvature and monotonically decreasing nature is meaningful when compared with data.
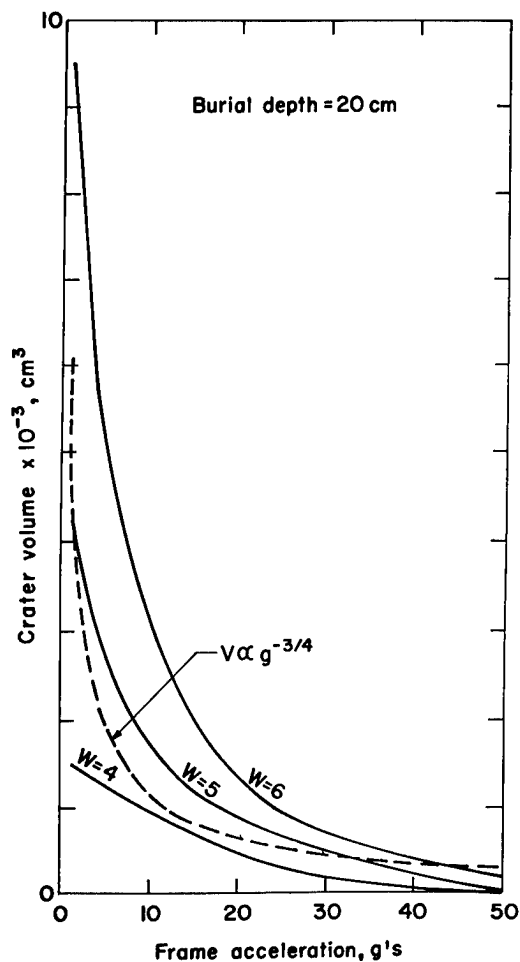
Figure 6 Results of experiments by
Viktorov and Stepenov show-
ing influence of increased
gravity on crater volume.

A cube-root or 3/10 prediction plotted in Figure 6 would be a horizontal line.
Crater-volume dependence on acceleration shown in Figure 6 is typical of results
obtained by Viktorov and Stepenov for the other crater dimensions, and for different
charge weights and burial depths. While scatter in the data of Viktorov and Stepenov
is relatively great, the influence of gravity in cratering has been unmistakably
shown. Since neither the cube-root nor 3/10 rule is capable of describing this in-
fluence, and since the fourth-root rule provides a meaningful qualitative description
it is felt that the experiments of Viktorov and Stepenov lend additional support to
the belief that fourth-root rules are the fundamental scaling laws for craters formed
by buried explosions.

Undoubtedly by this time our harried experimenter, obliged to perform two
cratering experiments with crater dimensions that must scale, will have reached the
height of his frustrations. Ericsson and Edin's experiments lend support to energy
scaling over mass scaling. Consequently, either cube-root or fourth-root scaling is
fundamental to cratering, depending on whether g is significant or not. Small-charge
data reveal that cube-root scaling is adequate. Nevada crater data from larger
charges indicate that cube-root scaling is inadequate. Nevada data, however, do not
confirm fourth-root scaling either. Lack of similarity among experiments apparently
accounts for these confusing results. Attempts to describe qualitatively the in-
fluence of possible sources contributing to nonsimilarity would suggest that each

has the effect of producing larger scaled crater dimensions for larger than for smaller explosions. Since Nevada crater data demonstrate this effect for fourth-root (but not cube-root) scaling, the implication is that fourth-root scaling is fundamental. Crater experiments of Viktorov and Stepenov in accelerated frames also lend support to fourth-root over cube-root scaling. An account of the adequacy of 1/3 scaling for small charges may be made by invoking atmospheric pressure as significant in cratering and as a source of similarity violation in fourth-root scaling. While no experimental data provide direct evidence to establish cube-root or fourth-root scaling as fundamental to cratering, a consistent qualitative explanation of all crater data is possible on the basis of similarity violations only if fourth-root scaling is assumed to be fundamental. It seems plausible that the experimenter would most likely choose fourth-root scaling to execute his experiments. Nevertheless, he would do so with considerable apprehension, for it is possible that all the results of experiments could be explained by the influence of certain variables not yet considered; for example, an anistropy resulting from variations in density, moisture content, or porosity with depth. Certainly he would prefer to see more experiments performed which would verify his inferences from current data and which would eliminate many of his doubts. Controlled laboratory experiments with small charges in homogeneous isotropic media to examine the influence of atmospheric pressure and the scaling nature of explosive parameters, ρ_x and q, would be informative. It would also be desirable to perform more cratering experiments such as those of Viktorov and Stepenov to investigate more closely the influence of g. In short, our experimenter would clearly like to see someone else attempt to perform his task from which, hopefully, he could make further deductions about scaling and fortify or alter appropriately his current concepts. If there were no time limit set for the execution of his obligation, the experimenter would certainly wait patiently for more experiments to be performed.

Chapter 5

SUMMARY AND CONCLUSIONS

Dimensional analyses have been performed on a number of physical variables thought to be significant in the phenomena of cratering by buried explosions. Gravitational acceleration has been both omitted and included in the analyses, and the explosion source has been described by both a mass and an energy dimension. Four different sets of scaling rules are obtained. When gravitational acceleration, g, is not included in the dimensional analyses, Lampson's[1] cube-root scaling is obtained when an explosion is characterized by a mass dimension, and Sachs'[20] cube-root scaling results when the explosion is characterized by an energy dimension. Under the conditions of all cratering experiments to date, it is not possible to distinguish between the "mass" scaling of Lampson and the "energy" scaling of Sachs. Air-blast experiments of Ericsson and Edin[21] indicate that, of these two cube-root rules, energy scaling is to be preferred.

When g is included in the dimensional analysis, then cube-root scaling is obtained for linear crater dimensions if the explosion is described by a mass dimension, but fourth-root scaling is obtained when the explosion is described by an energy dimension.

If the results of Ericsson and Edin can be extended to explosions underground, either Sachs' cube-root rule of Haskell's fourth-root rule is the fundamental scaling for cratering experiments; that is to say, if cratering experiments with buried explosions could be performed for which similitude was achieved, then one could establish unambiguously which of the two scaling rules is fundamental to the description of crater dimensions.

In the cratering experiments usually performed, it appears that violations of similarity requirements are severe enough to make impossible a direct answer as to which scaling is correct. Extensive crater data from explosions in alluvial soil ranging from 10^2 to 10^8 pounds (TNT) demonstrate lack of similarity among experiments. When crater dimensions are scaled by the cube-root rule, scaled dimensions are smaller for the larger explosions than for the smaller explosions, while crater dimensions scaled by the fourth-root rule are larger for larger explosions.

Results of the dimensional analysis provides some qualitative explanations of the possible sources contributing to the deviations from scaling rules because of similarity violations. Observed deviations from fourth-root scaling can be attributed to inability to scale any or all of the medium properties, ν, Y, and c, atmospheric pressure, k, and particularly charge radius, a.

Crater data from chemical explosions (10^2 to 10^6 pounds TNT) are scaled best by the approximate "overburden" rule. The "ad hoc" 3/10 rule appears to be an empirical demonstration of the lack of similarity among experiments. There appear to be no sound arguments which favor a 3/10 rule as being fundamental. The approximate "overburden" rule is a derivative of the dimensional analysis leading to fourth-root scaling. It offers some attraction because it is capable of describing the cube-root scaling observed for craters produced by small explosions (less than 10^2 pounds TNT) and the empirical 3/10 scaling for explosions of size 10^2 to 10^6 pounds. Also, for very large explosions, it reduces to the fourth-root rule, one of the two possible fundamental scalings. Inherent in this description by the "overburden" rule is the interpretation that lithostatic <u>and</u> atmospheric pressure are significant in cratering. At first sight, it may seem surprising that atmospheric pressure can be influential in determining a crater dimension. However, when it is recognized that almost all the cratering experiments in desert alluvium have been conducted at burial depths, where lithostatic is comparable to atmospheric pressure and where the contribution of atmospheric pressure to nonsimilarity is greatest, then the suspicion that atmospheric pressure may be significant is not so remote. For small explosions, 10^2 pounds TNT or less, atmospheric is considerably greater than lithostatic pressure and the "overburden" rule reduces to cube-root scaling in agreement with observations.

The 3/10 and the "overburden" rules cannot both be correct over an arbitrarily large range of explosion energies since, as we have seen, the "overburden" rule reduces to fourth-root scaling for very large explosions and deep burial depths. Differences in crater dimensions for a megaton explosion, when scaled from kiloton data by the two rules, can be substantial. As a result, it must be expected that the empirical 3/10 rule may have definite limitations in predicting crater dimensions for large nuclear explosions.

Evaluations of the various scaling rules by comparison of data scatter, or with the coefficient of determination, reveal that cube-root scaling is probably not valid, that fourth-root or "overburden" rules scale data best for explosions greater than 20 tons TNT, and that some difference probably exists in the relative cratering effectiveness of nuclear and chemical explosions. That fourth-root rules scale data better than cube-root or 3/10 rules for the larger explosions, as qualitatively predicted by dimensional analysis, lends some support to acceptance of fourth-root rules as the fundamental scaling of crater sizes and to the hypothesis that gravity is significant in cratering. Viktorov and Stepenov's[31] cratering experiments with small charges in accelerated frames have provided some direct evidence on the influence of gravity in cratering.

It must be emphasized that experimental evidence favoring fourth-root scaling is not definitive and that interpretation of the evidence in favor of a fourth-root rule is not conclusive. Uncertainty in interpretation is attributed to insufficient data and to the lack of similarity among experiments. The need for additional experiments is apparent. Some very-small-scale experiments not yet performed, but

which are suggested by the "overburden" rule, are cratering explosions in which the ambient "atmospheric" pressure is varied from vacuum to many bars. Also, the fourth-root rule, which requires that charge radius be proportional to the fourth root of charge energy, suggests experiments with explosives of differing composition.

Ability to obtain reliable estimates of crater dimension for large nuclear explosions in media where only chemical-explosion crater data are available depends on the resolution of the two basic questions, scaling and relative cratering effectiveness of nuclear compared to chemical explosions. The question of relative effectiveness is not currently answerable from available data because the answer depends on which scaling is chosen and the scaling question is not definitely resolved by data. The most direct way of determining nuclear-explosion effectiveness is to perform two identical experiments, one with nuclear and one with chemical explosions.

In future experiments, particularly with nuclear explosions, it is suggested that burial depths be governed by fourth-root rather than cube-root or 3/10 rules. By this criterion, data from similar or nearly similar experiments (according to the fourth-root rule) could be obtained which would hopefully provide some indication as to the validity or lack of validity of fourth-root scaling.

Until more data are obtained and until the basic questions are answered, the approximate "overburden" scaling rule, with nuclear and chemical explosions equivalent in effectiveness, is recommended as a guide in practical considerations.

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